Crowded pseudocompact Tychonoff spaces of cellularity at most the continuum are resolvable

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- 3 The van Mill's construction
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## Definitions

A crowded space is resolvable if it contains two disjoint dense sets and a space is pseudocompact if every continuous function with real values is bounded.

These two notions were introduced by Hewitt in the 1940's:

• E. Hewitt, *A problem of set-theoretic topology*, Duke Math. J. **10** (1943), 309–333.

• E. Hewitt, *Rings of real-valued continuous functions, I*, Trans. Amer. Math. Soc. (1948), 45–99.

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The notion of resolvability comes from a problem of Katětov:

Is there a Hausdorff crowded space *X* such that every function  $f: X \to \mathbb{R}$  is continuous at some point ?

• M. Katětov, *On topological spaces containing no disjoint dense sets*, Mat. Sib. 21 (1947), 3–12

and the next observation of Malykhin:

the answer is yes iff there exists a Hausdorff Baire irresolvable space

• V. I. Malykhin, On the resolvability of the product of two spaces, and a certain problem of Katětov, Dokl. Akad. Nauk SSSR 222 (1975), 725–729

So there are well known work about resolvability on Baire spaces and we know now that the answer to this problem is independent to *ZFC* 

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In the same paper they ask:

Is every Tychonoff crowded pseudocompact space resolvable?
W. W. Comfort and S. García-Ferreira *Resolvability: a selective survey and some new results*, Topology Appl. 74, (1996) 149–167

This is a natural question because every countable compact space is pseudocompact and every Tychonoff pseudocompact space is Baire

The first partial solution was given by van Mill who proved that every Tychonoff pseudocompact *ccc* space is c-resolvable

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## We may mention something about resolvability in ccc spaces.

Casarrubias-Segura, Hernández-Hernández and Tamariz-Mascarúa proved with MA that every  $T_2$  crowded Baire *ccc* space is  $\omega$ -resolvable

Dorantes-Aldama showed, assuming that c is less than the first weakly inaccessible cardinal, that every  $T_2$  crowded *ccc* Baire space is resolvable and the existence of a crowded *ccc* Baire space is equivalent to the existence of a measurable cardinal

For the following all the spaces considered are crowded and Tychonoff

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# The first step is standard

Define a tree of nonempty open sets of X such that:

1 
$$U_0 = \{X\},\$$

- 2  $\mathcal{U}_{\alpha}$  is cellular for every  $\alpha < \kappa$ , and
- 3 { $U \in U_{\alpha+1} : U \subseteq V$ } and { $cl(U) : U \in U_{\alpha+1}$  and  $U \subseteq V$ } are infinite maximal cellular families with respect to *V* for every  $\alpha < \kappa$ and  $V \in U_{\alpha}$ ,

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4 if 
$$\{U_{\xi} : \xi < \alpha\}$$
 is a chain, then  $cl(U_{\xi+1}) \subseteq U_{\xi}$ .

 $\kappa$  is the ordinal number where the construction stops

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# Define

1 
$$F_{\alpha} = \left[\bigcap_{\xi < \alpha} (\bigcup \mathcal{U}_{\xi})\right] \setminus (\bigcup \mathcal{U}_{\alpha})$$
 for every  $\alpha < \kappa$  and  $F_{\kappa} = \bigcap_{\xi < \kappa} (\bigcup \mathcal{U}_{\xi})$   
2  $\Lambda = \{\alpha \le \kappa : cof(\alpha) = \omega\}$ 

#### Observe that

 $\kappa \leq c(X)^+$ , and

if *X* is pseudocompact then  $F_{\alpha} \neq \emptyset$  for each  $\alpha \in \Lambda$ 

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Each open set of the tree is related to a function $f : X \to [0, 1]$ , so each $\alpha < \omega_1$ it is defined a function from X to a countable power of $[0, 1]$				o for of	

van Mill observes that every open set is eventually divided by the tree

Cantor sets appears after  $\omega$  divisions of an open set because of the pseudocompactness

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# When $c(X) > \omega$

The van Mill's way could lead us through uncountable powers and then we may lose metrizability. Fortunately some facts can be translated and used:

 if U ⊆ ⋃<sub>ξ≤α</sub> F<sub>ξ</sub> for some α < κ, then U is divided by the tree before α

2 by pseudocompactness, if α ∈ Λ and {U<sub>ξ</sub> ∈ U<sub>ξ</sub> : ξ < α} is a chain, then ( ∩<sub>ξ<α</sub> U<sub>ξ</sub>) ∩ F<sub>α</sub> ≠ Ø

### Definition

For each  $n \in \mathbb{N}$ , let

 $\Gamma_n = \left\{ f : \{0,1\}^n \to \bigcup_{\xi < \kappa} \mathcal{U}_{\xi} : f \text{ is injective and } f[\{0,1\}^n] \text{ is cellular } \right\}$ 

and let 
$$\Gamma = \bigcup_{n \in \mathbb{N}} \Gamma_n$$

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- Given  $n, m, f \in \Gamma_n$  and  $g \in \Gamma_m$  we will say that g extends f if n < m and the set  $\{g(y) : g(y) \subseteq f(x)\}$  is the set  $\{g(y) : y \text{ extends } x\}$  for every  $x \in \{0, 1\}^n$ .
- **2**  $f, g \in \Gamma$  are compatible if they are equal or if one of them extends the other.

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Let  $\alpha \in \Lambda$  and  $Y \subseteq X$ . Suppose that there are a strictly increasing sequence of ordinals  $(\alpha_n)_{n \in \mathbb{N}}$  cofinal in  $\alpha$  and a chain of compatible functions  $f_n : \{0,1\}^n \to \mathcal{U}_{\alpha_n}(Y)$ . Then, for every sequence of ordinals  $(\beta_n)_{n \in \mathbb{N}}$  cofinal in  $\alpha$  there is a strictly increasing function  $\phi : \mathbb{N} \to \mathbb{N}$ and a chain of compatible functions  $g_m : \{0,1\}^m \to \mathcal{U}_{\beta_{strin}}(Y)$ .

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#### Lemma

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Given $\alpha \in \Lambda$						
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Fix a strictly increasing sequence of ordinals $(\alpha_n)_{n \in \mathbb{N}}$ and define						

 $\mathbb{C}_{\alpha}(Y)$  as the set of all chains of compatible functions  $\{f_n \in \Gamma_n : n \in \mathbb{N}\}$  so that there is a strictly increasing function  $\phi : \mathbb{N} \to \mathbb{N}$  satisfying that  $f_n[\{0, 1\}^n] \subseteq \mathcal{U}_{\alpha_{\phi(n)}}(Y)$  for each  $n \in \mathbb{N}$ .

By the last lemma  $\mathbb{C}_{lpha}$  represent all the Cantor-like families in  $F_{lpha}$ 

Observe that  $|\mathbb{C}_{\alpha}(X)| \leq c(X)^{\omega}$ 

Given a chain  $\{f_n : n \in \mathbb{N}\} \in \mathbb{C}_{\alpha}(Y)$ , let  $f_{\{f_n:n \in \mathbb{N}\}} : \{0,1\}^{\omega} \to P(F_{\alpha})$  be the function defined by

$$f_{\{f_n:n\in\mathbb{N}\}}(x) = \left( \left(\bigcap_{n\in\mathbb{N}} f_n(x|_n)\right) \setminus int\left(\bigcap_{n\in\mathbb{N}} f_n(x|_n)\right) \right)$$

### for each $x \in \{0, 1\}^{\omega}$

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Fix a strictly increasing sequence of ordinals  $(\alpha_n)_{n \in \mathbb{N}}$  and define  $\mathbb{C}_{\alpha}(Y)$  as the set of all chains of compatible functions  $\{f_n \in \Gamma_n : n \in \mathbb{N}\}$  so that there is a strictly increasing function  $\phi : \mathbb{N} \to \mathbb{N}$  satisfying that  $f_n[\{0, 1\}^n] \subseteq \mathcal{U}_{\alpha_{\phi(n)}}(Y)$  for each  $n \in \mathbb{N}$ .

By the last lemma  $\mathbb{C}_{\alpha}$  represent all the Cantor-like families in  $F_{\alpha}$ 

Observe that  $|\mathbb{C}_{\alpha}(X)| \leq c(X)^{\omega}$ 

Given a chain  $\{f_n : n \in \mathbb{N}\} \in \mathbb{C}_{\alpha}(Y)$ , let  $f_{\{f_n:n \in \mathbb{N}\}} : \{0,1\}^{\omega} \to P(F_{\alpha})$  be the function defined by

$$f_{\{f_n:n\in\mathbb{N}\}}(x) = \left( \left(\bigcap_{n\in\mathbb{N}} f_n(x|_n)\right) \setminus int\left(\bigcap_{n\in\mathbb{N}} f_n(x|_n)\right) \right)$$

for each  $x \in \{0, 1\}^{\omega}$ 

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	Trees of open refinements	van Mill's construction	Main ideas	Bibliography.
Cruc	ial Facts:			

 $\{t_n : n \in \mathbb{N}\} \in \mathbb{C}_{\alpha}(O)$ , then  $t_{\{t_n:n \in \mathbb{N}\}}(x) \cap cl(O) \neq \emptyset$  for every  $x \in \{0,1\}^{\omega}$ 

#### Fact III

Suppose that X is pseudocompact and  $c(X) \leq c$ . If O is an open subset contained in  $\bigcup_{\xi \in \Lambda_{\alpha}} F_{\xi}$  satisfying that  $\mathcal{U}_{\xi}(V) \neq \emptyset$  for every nonempty open  $V \subseteq O$  and every  $\xi < \alpha$ , then O is resolvable (*c*-resolvable)



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Cruc	cial Facts:		

## Fact I

# By the definition of the tree, if $x, y \in \{0, 1\}^{\omega}$ , then

$$f_{\{f_n:n\in\mathbb{N}\}}(x)\cap f_{\{f_n:n\in\mathbb{N}\}}(y)=\emptyset$$
 iff  $x\neq y$ 

#### Fact I

Let *X* be pseudocompact and let *O* be an open set. If  $\{f_n : n \in \mathbb{N}\} \in \mathbb{C}_{\alpha}(O)$ , then  $f_{\{f_n:n\in\mathbb{N}\}}(x) \cap cl(O) \neq \emptyset$  for every  $x \in \{0,1\}^{\omega}$ 

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- 3 The van Mill's construction
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Main ideas

# Sketch of the proof

**1** Consider the dense subspace  $\bigcup_{\alpha \in \Lambda} F_{\alpha}$ 

- **2** by transfinite induction on  $\Lambda$  with the order type will be showed that  $int(\bigcup_{\xi \in \Lambda_{\alpha}} F_{\xi})$  is resolvable when  $int(\bigcup_{\xi \in \Lambda_{\alpha}} F_{\xi}) \neq \emptyset$
- **3** if  $int(F_{\alpha}) \neq \emptyset$ , then  $int(F_{\alpha})$  is resolvable
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The proof

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	Trees of open refinements			The proof	Bibliography.
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