Dynamical systems on compact metric countable spaces

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Content

Ellis semigroup

In our dynamical systems (X, f), X will be a compact metric space and $f : X \to X$ a continuous function.

For $n \in \mathbb{N}$, f^n denotes the *n-iterate* of a continuous function $f : X \to X$.

Given a dynamical system (X, f), the *Ellis semigroup*, denoted by $E(X, f)$, is the pointwise closure of $\{f^n : n \in \mathbb{N}\}$ in the compact space X^X with composition of functions as its algebraic operation.

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Old Problem

Given a compact metric space X, when are the functions of

$$E(X,f) \setminus \{f^n : n \in \mathbb{N}\} = E(X,f)^*$$

either all continuous or all discontinuous?

The answer is yes when X is a convergent sequence with its limit point.

When $X = [0,1]$, P. Szuca (2013) showed that if $f : [0,1] \to [0,1]$ is a continuous surjection and if $E([0,1],f)^*$ contains a continuous function, then all functions of $E(X,f)^*$ are continuous.

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Countable spaces

Theorem, 2015

Let (X, f) be a dynamical system such that X is a compact metric countable space and every accumulation point of X is periodic. Then either all function of $E(X, f)^*$ are continuous or all of them are discontinuous.

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Let (X, f) be a dynamical system such that X is a compact metric countable space. If X has finitely many accumulation points, then either all function of $E(X, f)^*$ are continuous or all of them are discontinuous.

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There is a dynamical system (X, f) where X is a compact metric countable space such that the orbit of each accumulation point is finite and that there are $f_0, f_1 \in E(X, f)^*$ so that f_0 is continuous on X and f_1 is discontinuous on X.

The space X is the countable ordinal space $\omega^2 + 1$ which is identified with a suitable subspace of $\mathbb{R}$.

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Cardinality

In this talk we were interested on the cardinality of the Ellis semigroup $E(X, f)$. The work of A. Köhler (1995) and M. E. Glasner and Megrehisvili (2006) contain very interesting results about the cardinality of $E(X, f)$. Indeed, M. E. Glasner and Megrehisvili stablished the *Bourgain-Fremlin-Talagrand dichotomy for dynamical systems*: Either $|E(X, f)| \leq \mathfrak{c}$ or $E(X, f)$ contains a copy of $\beta\mathbb{N}$.

We willl be mostly concerned with countable compact metrizable spaces. In this case, it is evident that $|E(X, f)| \leq \mathfrak{c}$. Moreover, since $E(X, f)$ is a separable metric space, then $E(X, f)$ is either countable or has cardinality $\mathfrak{c}$.

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Ultrafilters

$\beta(\mathbb{N})$ will denote the set of all ultrafilters on $\mathbb{N}$ y $\mathbb{N}^* = \beta(\mathbb{N}) \setminus \mathbb{N}$ will be the set of all free ultrafilters on $\mathbb{N}$.

$\beta(\mathbb{N})$ is the Stone-Čech compactification of the natural numbers $\mathbb{N}$ with the discrete topology.

If $A \subseteq \mathbb{N}$, then $\hat{A} = \{p \in \beta(\mathbb{N}) : A \in p\}$ is a basic open subset of $\beta(\mathbb{N})$ and $A^* = \hat{A} \setminus \mathbb{N}$ is a basic open subset of $\mathbb{N}^*$.

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p-limit points

Definition [Several mathematicians]

Let $p \in \mathbb{N}^*$. Let X a space and $(x_n)_{n \in \mathbb{N}}$ a sequence in X. We say that $x \in X$ is a *p-limit* of $(x_n)_{n \in \mathbb{N}}$ if for every neighborhood V of x we have that $\{n \in \mathbb{N} : x_n \in V\} \in p$.

We write $x = p - lim_{n \to \infty} x_n$.

$x \in X$ is an accumulation point of $\{x_n : n \in \mathbb{N}\}$ iff there is $p \in \mathbb{N}^*$ such that $x = p - lim_{n \to \infty} x_n$.

In a compact space, every sequence has a p-limit point for every $p \in \mathbb{N}^*$.

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Let $X = [0,1]$ and let $f : [0,1] \to [0,1]$ any continuous function such that $f(0) = 0$, $f(1) = 1$ and $f(t) < 1$ for all $t \in (0,1)$. Then, f is a continuous function such that $f^p[[0,1)] = 0$ and $f^p(1) = 1$, for each $p \in \mathbb{N}^*$. Therefore, f^p is not continuous at 1, for any $p \in \mathbb{N}^*$.

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$$E(X, f) = \{f^p : p \in \beta(\mathbb{N})\}$$

and

$$E(X, f)^* \subseteq \{f^p : p \in \mathbb{N}^*\}$$

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$$p + n = p - \lim_{m \to \infty} (m + n)$$

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Now, if $p, q \in \beta(\mathbb{N})$, then we define

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For $p \in \beta(\mathbb{N})$ and $n \in \mathbb{N}$, we define

$$p + n = p - \lim_{m\to\infty}(m + n)$$

Folklore

Now, if $p, q \in \beta(\mathbb{N})$, then we define

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Theorem, Folklore

If (X, f) is a dynamical system, then,

$$f^p \circ f^q = f^{q+p},$$

for every $p, q \in \beta(\mathbb{N})$.

Notice that if f^p is continuous for some $p \in \mathbb{N}^*$, then f^{p+n} is also continuous for all $n \in \mathbb{N}$.

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Content

Cardinality

Theorem

Let (X, f) be a dynamical system. Then $E(X, f)$ is finite iff there exist $M > 0$ such that $|\mathcal{O}_f(x)| < M$ for each $x \in X$.

It is noteworthy that $E(X, f)^*$ could be finite and $E(X, f)$ could be infinite. For instance, if X is a convergent sequence with its limit point and f is the shift function, then $E(X, f)$ is infinite and $E(X, f)^*$ has only one point.

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For a dynamical system (X, f), the ω*–limit set* of $x \in X$, denoted by $\omega_f(x)$, is the set of points $y \in X$ for which there exists an increasing sequence $(n_k)_{k\in\mathbb{N}}$ such that $f^{n_k}(x) \to y$.

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For a dynamical system (X, f), let P_f denote the set of all periods of the periodic points of (X, f) which are accumulation points.

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Countable case

Theorem, S. Mazurkiewicz and W. Sierpinski, 1920

Every compact metric countable space is homeomorphic to a countable ordinal with the order topology.

In what follows, our phase space will be the compact metric space $\omega^\alpha + 1$ where α is a countable ordinal.

For our convenience, X' will denote the set of limit points of X, d will stand for the unique point of $\omega^\alpha + 1$ of CB-rank α and $\{d_n : n \in \mathbb{N}\}$ will be the collection of all its points with CB-rank equal to $\alpha - 1$.

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Let $(\omega^\alpha + 1, f)$ be a dynamical system with $\alpha \geq 1$ a countable successor ordinal, such that there exists $w \in \omega^\alpha + 1$ with a dense orbit. Then the following conditions hold:

- (i) $f(y)$ is a limit point for every $y \in (\omega^\alpha + 1)'$.
- (ii) The range of f is $\omega^\alpha + 1 \setminus \{w\}$.
- (iii) If $x \in (\omega^\alpha + 1)'$, then $\emptyset \neq f^{-1}(x) \subseteq (\omega^\alpha + 1)'$.
- (iv) $1 \leq CB(f(d))$.

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Let $(\omega^2 + 1, f)$ be a dynamical system such that there exists $w \in \omega^2 + 1$ with a dense orbit. Then f^p is continuous, for every $p \in \mathbb{N}^*$, and $E^*(\omega^2 + 1, f)$ is countable.

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There is a continuous function $f : \omega^3 + 1 \to \omega^3 + 1$ such that

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Is there a dynamical system (X, f) such that X is connected and there are two functions $f_0, f_1 \in E^*(X, f)$ such that f_0 is continuous and f_1 is discontinuous?

Given a dynamical system $(\omega^\alpha + 1, f)$ with dense orbit, where $\alpha > 3$, is $E(\omega^\alpha + 1, f)$ always countable?

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