Toposym 2022
Book of Abstracts

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Preface

The first Topological Symposium took place in 1961 at a time when the world was still painfully divided during the Cold War. Communication across the border was difficult, although the situation had improved somewhat over the previous decade. In this state of things Eduard Čech decided to organize an event which would bring together mathematicians from the East and the West. It was an enormous effort on his part and, alas, he was not able to see the fruits of it – he died in 1960. However his efforts were not in vain. His students and colleagues managed to finish what he started and in 1961, 147 mathematicians gathered in Prague for a week devoted to topology. This has started a tradition that every five years mathematicians from all over the world, interested in diverse areas of topology, come to meet in Prague.

The Thirteenth Symposium on General Topology and its Relations to Modern Analysis and Algebra – Toposym 2022 – is held in Prague on July 25–29, 2022. The symposium is organized by the Faculty of Mathematics and Physics of the Charles University in Prague, the Institute of Mathematics of the Czech Academy of Sciences, the Faculty of Information Technology of the Czech Technical University in Prague. The meeting is held in the lecture halls of the Faculty of Information Technology, Czech Technical University in Prague. The symposium is attended by about 120 mathematicians from multiple countries. The program of the symposium consists of 24 invited lectures delivered by mathematicians selected by the scientific committee, an opening lecture delivered by Jan van Mill, more than 60 selected contributed talks, and 4 poster presentations.

Adam Bartoš and David Chodounský

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Invited speakers

Plenary

Taras Banakh
Alan Dow
Eli Glasner
Michael Hrušák
Ondřej Kurka
Jan van Mill
Piotr Oprocha
Vladimir Pestov
Christian Rosendal
Mikhail Tkachenko
Stevo Todorčević
Boaz Tsaban

Semi-plenary

Sergey Antonyan
Antonio Avilés
Dana Bartošová
Will Brian
Dikran Dikranjan
Rodrigo Hernández-Gutiérrez
Alejandro Illanes
István Juhász
Olena Karlova
Piotr Koszmider
Aleksandra Kwiatkowska
Alexander Shibakov
Lyubomyr Zdomskyy

PLENARY TALKS

The program of Toposym 2022 contains 11 invited plenary talks delivered by mathematicians selected by the scientific committee of the symposium and an opening lecture delivered by Jan van Mill, the chair of the scientific committee. All plenary talks are 50 minutes long.

Automatic continuity of measurable homomorphisms on Čech-complete topological groups

Taras Banakh

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The problem of automatic continuity of measurable homomorphisms traces its history back to Cauchy who proved in 1821 that continuous additive real functions are linear and asked about conditions implying the continuity of additive functions. In the very first issue of Fundamenta Mathematice (1920), three papers (of Banach, Sierpiński and Steinhaus) were dedicated to the proof of the continuity of Lebesgue measurable additive real functions. Later those results were extended by Weil, Pettis, Christensen and other great mathematicians of XX century who substantially contributed to the theory of automatic continuity. In the talk we discuss the resent progress in extending classical results on automatic continuity beyond the class of Polish groups. In particular, we present a new

Theorem. *A homomorphism $h : X \to Y$ from a Čech-complete topological group X to a topological group Y is continuous iff h is Borel-measurable iff h is universally measurable iff h is universally BP-measurable. If X is ω-narrow (and locally compact), then h is continuous iff h is BP-measurable (iff h is Haar-measurable).*

This theorem extends a recent result of Rosendal (2019) on the continuity of universally measurable homomorphisms between Polish groups, and an old result of Kleppner (1989–91) on the continuity of Haar-measurable homomorphisms between locally compact groups. Details can be seen at (`arxiv.org/abs/2206.02481`).

S and Moore–Mrowka spaces under really not CH

Alan Dow[1], Saharon Shelah[2]*

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A Moore–Mrowka space is a compact non-sequential space of countable tightness. Their existence is independent of CH and of MA + $\mathfrak{c} = \omega_2$. This got me to wondering about $\mathfrak{c} > \omega_2$. It then was natural to wonder the same thing about S spaces since I have always felt that the Moore–Mrowka question relied heavily on developments and techniques from the S space problem. Using methods, primarily from Todorcevic [1983], but also from Abraham–Shelah [1981] and Abraham–Rubin–Shelah [1985], we answer the problem for S spaces and make some progress for Moore–Mrowka.

[1]The first author was partially supported by Topology and its Applications.
[2]The second author was supported by NSF-DMS 1101597.

Todorčević' trichotomy and a hierarchy in the class of tame dynamical systems

Eli Glasner

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I will briefly review the newly developed theory of tame dynamical systems and then show how Todorčević' trichotomy in the class of separable Rosenthal compacta is reflected as a hierarchy in the class of tame dynamical systems (X, T) according to the topological properties of their enveloping semigroups $E(X)$. More precisely, I will define the classes

$$\mathrm{Tame}_{\mathbf{2}} \subset \mathrm{Tame}_{\mathbf{1}} \subset \mathrm{Tame},$$

where $\mathrm{Tame}_{\mathbf{1}}$ is the proper subclass of tame systems with first countable $E(X)$, and $\mathrm{Tame}_{\mathbf{2}}$ is its proper subclass consisting of systems with hereditarily separable $E(X)$. Some general properties of these classes will be discussed and I will exhibit some examples to illustrate these properties. This is a joint work with Michael Megrelishvili.

Ultrafilters and countably compact groups

Michael Hrušák

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We shall discuss our recent joint result with van Mill, Ramos-Garcia and Shelah – a ZFC construction of a countably compact topological group without non-trivial convergent sequences – and related properties of ultrafilters on countable sets.

Complexity of distances, reducibility and universality

Ondřej Kurka

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We introduce and study the notion of Borel reducibility between pseudometrics on standard Borel spaces, which is a generalization of the famous notion of Borel reducibility between equivalence relations.

The central object of our investigations is the Gromov–Hausdorff distance, which turns out to be equally complex as several other distances between metric or Banach spaces, such as the Kadets distance or the Banach–Mazur distance. Next, we consider the notion of an orbit pseudometric and provide a continuous version of the well-known result of Clemens, Gao and Kechris that the relation of isometry of Polish metric spaces is bireducible with a universal orbit equivalence relation.

The present results come from the collaboration with Marek Cúth and Michal Doucha; see [1, 2, 3].

[1] M. Cúth, M. Doucha, and O. Kurka, *Complexity of distances between metric and banach spaces: Theory of generalized analytic equivalence relations*, J. Math. Logic (to appear).

[2] M. Cúth, M. Doucha, and O. Kurka, *Complexity of distances between metric and banach spaces: Reductions of distances between metric and banach spaces*, Israel J. Math., 248 (2022), pp. 383–439.

[3] O. Kurka, *Orbit pseudometrics and a universality property of the Gromov–Hausdorff distance*, arXiv:2204.08375.

Many weak P-sets

Jan van Mill, Alan Dow*[1]

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It is known that for each continuous image of $\mathbb{N}^*$, there is a nowhere dense weak P-set of $\mathbb{N}^*$ that maps irreducibly onto it. We generalize this for every compact space of weight at most $\mathfrak{c}$. This allows us to show that there is a weak P-set in $\mathbb{N}^*$ which is homeomorphic to $\mathbb{N}^*$. This generalizes a result of the second-named author and answers a problem posed before 1990.

[1] The second author was supported by NSF grant No. DMS-1501506.

On pseudoarc and dynamics

Piotr Oprocha[1]

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The pseudoarc is an intriguing continuum that was discovered by Knaster 100 years ago. While its structure is very complicated, Bing proved that from topological perspective, pseudoarc is the typical continuum to encounter in the plane.

In this talk I will present selected results connecting pseudoarc as topological object with selected properties of dynamical systems.

[1]P.O. was partially supported by National Science Centre, Poland (NCN), grant no. 2019/35/B/ST1/02239.

On the problem of amenability of groups of maps

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Given a compact space X and a compact group K, is the group $C(X,K)$ of all continuous maps with the topology of uniform convergence amenable? (Recall that a topological group is amenable if every continuous action of the group on a compact space admits an invariant regular probability measure.)

The only case where the answer is quite trivially "yes" is where $\dim X = 0$. Already in the one-dimensional case of $X = [0,1]$ or $\mathbb{S}^1$, an affirmative answer, due to Marie-Paule Malliavin and Paul Malliavin (1992), requires using a subtle machinery of Wiener measures and going to maps of a higher smoothness class. Beyond those two settings, nothing seems to be known in the continuous case.

Actually, one needs to look at the higher smoothness degree maps also in order to understand the motivation of the question, which comes from mathematical physics. The analysis of the problem shows that perhaps the property needed is not exactly amenability, but one of its variants which is equivalent to amenability in the locally compact case: skew-amenability.

We will discuss the motivation, the background, the techniques, known results and open questions, reviewing, in addition to the above mentioned work, some recent results by the speaker and also by Kate Juschenko and F. Martin Schneider.

[1] The author was partially supported by FAPESQ/CNPq DCR-A fellowship.

Amenability, optimal transport and abstract ergodic theorems

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Using tools from the theory of optimal transport, four results concerning isometric actions of amenable topological groups with potentially unbounded orbits are established. Specifically, consider an amenable topological group G with no non-trivial homomorphisms to $\mathbb{R}$. If d is a compatible left-invariant metric on G, $E \subseteq G$ is a finite subset and $\epsilon > 0$, there is a finitely supported probability measure β on G so that

$$\max_{g,h \in E} \mathsf{W}(\beta g, \beta h) < \epsilon,$$

where W denotes the *Wasserstein* or *optimal transport* distance between probability measures on the metric space (G, d). When d is the word metric on a finitely generated group G, this strengthens a well known theorem of H. Reiter. Furthermore, when G is locally compact second countable, β may be replaced by an appropriate probability density $f \in L^1(G)$.

Also, when $G \curvearrowright X$ is a continuous isometric action on a metric space, the space of Lipschitz functions on the quotient $X /\!\!/ G$ is isometrically isomorphic to a 1-complemented subspace of the Lipschitz functions on X. And finally every continuous affine isometric action of G on a Banach space has a canonical invariant linear subspace. These results generalise previous theorems due to Schneider–Thom and Cúth–Doucha.

[1]Research partially supported by NSF award DMS 2204849.

On stability and weight of Lindelöf P-groups

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If every G_δ-set in a space X is open, we say that it is a *P-space*. Similarly, a topological group is a *P-group* if it is a P-space. The class of Lindelöf P-groups, which is similar to the class of compact topological groups in terms of permanence properties, is the focus of our presentation.

First, we consider the question of whether every Lindelöf P-group is τ-stable, for a given infinite cardinal τ. It is well known that the answer is affirmative for $\tau = \aleph_0$ and $\tau = \aleph_1$. We extend this conclusion to a proper class of cardinals τ, including those satisfying the equality $\tau^\omega = \tau$. We deduce, for example, that every Lindelöf P-group is $\aleph_n$-stable, for each $n \in \omega$.

Second, we look at the problem of estimating a gap between the weight and i-weight of a Lindelöf P-group G. If the cardinal $\tau = iw(G)$ is either $\aleph_n$ for some $n \in \omega$ or fulfills $\tau^\omega = \tau$, it is not difficult to demonstrate that the two cardinal functions coincide. In general, however, a gap can be quite big. According to our best knowledge, if $iw(G) = \aleph_\omega$, then $w(G) < \aleph_{\omega_4}$ and $w(G) \leq (\tau_\omega)^\omega$.

Our entire analysis is based on the family $[\tau]^\omega$ of countable subsets of an uncountable cardinal τ partially ordered by inclusion.

[1]The author was partially supported by the CONACyT of Mexico, grant FORDECYT-PRONACES/64356/2020.

Generic metrizability

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We present a method for proving that a given compact space contains a dense metrizable subspace. The method uses the language of Forcing but it can be reformulated in terms of generic points and the regular-open algebra of a given compact space. We will present some applications of the method solving some open problems from the literature. However, we will also show that the method gives a unified approach to some of the other well known theorems of this type found in the literature.

Selection principles and omission of intervals

Boaz Tsaban

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I will provide an overview of *omission of intervals*, a method that I developed for constructing subsets of the real line with extraordinary combinatorial properties, and answering some classic questions. These questions are best viewed in the unified framework of Selection Principles, a central topic in general and set theoretic topology (with applications that go far beyond my talk).

Additional mathematicians have, recently, used this method to obtain new insights on the real line, and settled some of the oldest and most important problems in the realm of Selection Principles. I will mention some of these breakthroughs.

SEMI-PLENARY TALKS

The program of Toposym 2022 contains 13 invited semi-plenary talks delivered by mathematicians selected by the scientific committee of the symposium. All semi-plenary talks are 40 minutes long, presented in two parallel sessions.

Some hyperspaces of compact convex sets and their orbit spaces

Sergey Antonyan[1]

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Let $cc(\mathbb{R}^n)$ denote the hyperspace of all nonempty compact convex subsets of the Euclidean space $\mathbb{R}^n$ endowed with the Hausdorff metric, and let $cc(\mathbb{B}^n) = \{A \in cc(\mathbb{R}^n) \mid A \subset \mathbb{B}^n\}$, where $\mathbb{B}^n$ is the closed unit ball of $\mathbb{R}^n$. In this talk we will describe the topological structure of several geometrically defined subspaces of $cc(\mathbb{R}^n)$ and $cc(\mathbb{B}^n)$ and their orbit spaces under the natural action of the orthogonal group $O(n)$. Among them are the hyperspaces $cb(\mathbb{B}^n) = \{A \in cc(\mathbb{B}^n) \mid Int\, A \neq \emptyset\}$ and $\check{c}b(\mathbb{B}^n) = \{A \in cb(\mathbb{B}^n) \mid \check{C}(A) = \mathbb{B}^n\}$, where $\check{C}(A)$ denotes the circumball of A. We will also introduce some new models for the Banach–Mazur compacta $BM(n)$ as a by-product. Related open problems will be discussed.

[1]The author's research was supported by XATU (Xi'an Technological University) and grant IN-101420 from PAPIIT (UNAM).

Compact spaces associated to Banach lattices

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Given a vector x of a Banach lattice X, the ideal generated by x is canonically identified with a $C(K)$ space. In this way, we associate compact spaces to Banach lattices, and new interesting classes of compact spaces arise. In particular we shall focus on the compact spaces linked to separable Banach lattices.

[1]Reserach supported by Fundación Séneca (Murcia) [20797/PI/18], AEI (Spain) / FEDER,UE (MTM2017-86182-P first three authors and MTM2016-76808-P and MTM2016-75196-P last author). A. Rueda Zoca also supported by Junta de Andalucía (A-FQM-484-UGR18 and FQM-0185). P. Tradacete also supported by "Severo Ochoa Programme for Centres of Excellence in R&D" (CEX2019-000904-S)

Model theory in Ramsey theory

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Ramsey theory for finite structures has seen rich development in the recent decades. We will talk about two different directions we have taken to study interlacement of Ramsey theory and model theory. In a recent work with Lynn Scow, we take a new perspective on Scow's notion of semi-retractions, which relates two structures in (typically) different languages and we show the optimal conditions under which semi-retractions transfer Ramsey properties. We will note that semi-retractions have a natural relationship with a categorical notion of pre-adjunctions.

With Mirna Džamonja, Rehana Patel, and Lynn Scow, we investigate big Ramsey degrees of countable ultraproducts of finite structures with respect to internal and external colourings. While we get a general positive result for internal colourings, our results for external colourings thus far depend on the Continuum Hypothesis.

[1]The first author was partially supported by an NSF grant DMS-1953955. Parts of the project were obtained as during an AIM SQuaRE

Partitioning the real line into Borel sets

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For which infinite cardinals κ is there a partition of the real line $\mathbb{R}$ into precisely κ Borel sets?

Hausdorff famously proved that there is a partition of $\mathbb{R}$ into $\aleph_1$ Borel sets. The main theorem presented in this talk is that, other than Hausdorff's result, the spectrum of possible sizes of partitions of $\mathbb{R}$ into Borel sets can be fairly arbitrary. For example, given any $A \subseteq \omega$ with $0, 1 \in A$, there is a forcing extension in which $A = \{n$: there is a partition of $\mathbb{R}$ into $\aleph_n$ Borel sets$\}$.

We also look at the corresponding question for partitions of $\mathbb{R}$ into closed sets. We show that, like with partitions into Borel sets, the set of all uncountable κ such that there is a partition of $\mathbb{R}$ into precisely κ closed sets can be fairly arbitrary.

Entropy of amenable monoid actions

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For a right action $K \overset{\rho}{\curvearrowleft} S$ of a cancellative right amenable monoid S on a compact Hausdorff space K, we build its *Ore colocalization* $K^* \overset{\rho^*}{\curvearrowleft} G$, where K^* is a compact space and G is the group of left fractions of S. This construction preserves the topological entropy (i.e., $h_{\mathrm{top}}(\rho^*) = h_{\mathrm{top}}(\rho)$) and linearity of the action.

Similarly, for a left linear action $S \overset{\lambda}{\curvearrowright} X$ on a discrete Abelian group X, we construct its *Ore localization* $G \overset{\lambda^*}{\curvearrowright} X^*$, which is linear and preserves the algebraic entropy h_{alg} (i.e., $h_{\mathrm{alg}}(\lambda^*) = h_{\mathrm{alg}}(\lambda)$). Moreover, if $K \overset{\rho}{\curvearrowleft} S$ a right linear action with K a compact Abelian group and $S \overset{\rho^\wedge}{\curvearrowright} X$ is the dual left action on the discrete Pontryagin dual $X := K^\wedge$, then the Ore localization of $\rho^\wedge$ is conjugated to dual of the Ore colocalization $K^* \overset{\rho}{\curvearrowleft} G$. Using this fact, we prove the useful equality $h_{\mathrm{top}}(\rho) = h_{\mathrm{alg}}(\rho^\wedge)$, known also as *Bridge Theorem*.

We obtain an *Addition Theorem for* h_{top} (i.e., for a linear action $K \overset{\rho}{\curvearrowleft} S$ on a compact group K, a ρ-invariant closed subgroup H of K and the left cosets space K/H, $h_{\mathrm{top}}(\rho) = h_{\mathrm{top}}(\rho_H) + h_{\mathrm{top}}(\rho_{K/H})$), as well as a similar *Addition Theorem for* h_{alg}.

Hyperspaces of Erdős space

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Erdős space $\mathfrak{E}$ and complete Erdős space $\mathfrak{E}_c$ are two classical examples in dimension theory that have appeared in other contexts. For example, the set of endpoints of the Lelek fan is homeomorphic to $\mathfrak{E}_c$ (Kawamura, Oversteegen and Tymchatyn, 1996), and the set of self-homeomorphisms of the plane that fix a given countable dense set is homeomorphic to $\mathfrak{E}$ (Dijkstra and van Mill, 2010).

My Ph.D. student Alfredo Zaragoza started the study of the Vietoris hyperspaces of $\mathfrak{E}$ and $\mathfrak{E}_c$. Jointly with Alfredo, we found topological characterizations of the space $\mathfrak{E}_c \times \mathbb{Q}$, where $\mathbb{Q}$ is the space of rational numbers. Using these characterizations we were able to prove that the Vietoris hyperspace of finite sets of $\mathfrak{E}_c$ is homeomorphic to $\mathfrak{E}_c \times \mathbb{Q}$.

In this talk I will introduce these Erdős spaces, present our results and talk about some problems that are still open.

Embeddings of the pseudo-arc into some spaces

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A continuum is a compact connected metric space with more than one point. A continuum is hereditarily indecomposable if for every two subcontinua, either they are disjoint or one is contained in the other. The pseudo-arc is the simplest hereditarily indecomposable continuum. There are many reasons for considering the pseudo-arc as one of the most interesting continua.

One important topic in the study of the pseudo-arc is to determine how pseudo-arcs behave inside certain spaces.

In this talk we consider problems related to embeddings of the pseudo-arc in the following spaces: the Euclidean n-dimensional space, the product of two pseudo-arcs, the product of the pseudo-arc with the interval $[0, 1]$, the hyperspace of subcontinua of the pseudo-arc.

The double density spectrum of a topological space

István Juhász

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The set of densities of *all* dense subspaces of a topological space X is called the *double density spectrum* of X and is denoted by $dd(X)$.

We improve an earlier result by showing that $dd(X)$ is always ω-closed (i.e. countably closed) if X is Hausdorff. We characterize the double density spectra of Hausdorff and of regular spaces:

Let S be a non-empty set of infinite cardinals. Then

(1) $S = dd(X)$ for a Hausdorff space X if and only if S is ω-closed and $\sup S \leq 2^{2^{\min S}}$;

(2) $S = dd(X)$ for a regular space X if and only if S is ω-closed and $\sup S \leq 2^{\min S}$.

We do not have a characterization of the double density spectra of compact spaces but give some non-trivial consistency results concerning them:

(1) If $\kappa = cf(\kappa)$ embeds in $\mathcal{P}(\omega)/\mathrm{fin}$ and S is a set of uncountable regular cardinals $< \kappa$ with $|S| < \min S$, then there is a compactum C such that $\{\omega, \kappa\} \cup S \subset dd(C)$, moreover $\lambda \notin dd(C)$ whenever $|S| + \omega < cf(\lambda) < \kappa$ and $cf(\lambda) \notin S$.

(2) It is consistent to have a separable compactum C such that $dd(C)$ is not ω_1-closed.

This is joint work with J. van Mill, L. Soukup, and Z. Szentmiklóssy.

Baire-one functions on topological spaces: some recent results and open questions

Olena Karlova

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In this talk we will give an overview of relations between the following classes of functions defined on general topological spaces:

- Baire 1,
- F_σ-measurable,
- (functionally) fragmented,
- homotopic Baire 1,
- separately continuous,

with emphasis to more recent results and open questions.

New applications of Ψ-spaces in analysis

Piotr Koszmider[1]

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$\Psi_{\mathcal{A}}$ stands for the standard locally compact noncompact Ψ-space induced by an almost disjoint family $\mathcal{A}$. I will review a few new applications of Ψ-spaces in functional analysis. The first one involves $\mathbb{N} \times \mathbb{N}$ matrices acting on $C_0(\Psi_{\mathcal{A}})$ which generalize partitioners of almost disjont families or clopen subsets of $\Psi_{\mathcal{A}}$ and provide new examples in algebras of operators ([1]). The second one involves continuous functions from $\Psi_{\mathcal{A}}$ into 2×2 matrices with pointwise noncommutative multiplication and concerns C*-algebras ([2]). The third one is an equivalent renorming of $C_0(\Psi_{\mathcal{A}})$ which becomes the first example of a nonseparable Banach space where the unit sphere does not admit an uncountable subset X such that $\|x - y\| > 1$ for all distinct $x, y \in X$ ([3]). All these applications require special combinatorial properties of the almost disjoint families.

[1] P. Koszmider and N. J. Laustsen, *A Banach space induced by an almost disjoint family, admitting only few operators and decompositions*, Adv. Math., 381 (2021), paper no. 107613, 39 pages.

[2] O. Guzmán, M. Hrušák, and P. Koszmider, *On* $\mathbb{R}$*-embeddability of almost disjoint families and Akemann-Doner* C**-algebras*, Fund. Math., 254 (2021), pp. 15–47.

[3] P. Koszmider, *Banach spaces in which large subsets of spheres concentrate.* arXiv:2104.05335.

[1] The author was partially supported by the NCN (National Science Centre, Poland) research grant no. 2020/37/B/ST1/02613

The generic continuum approximated by finite graphs with confluent epimorphisms

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Irwin and Solecki introduced a projective Fraïssé limit, a dual concept to the (injective) Fraïssé limit from model theory, which they used to construct the pseudo-arc. For this they considered the class of finite linear (combinatorial) graphs together with epimorphisms preserving the edge relation, and showed that the topological realization of its Fraïssé limit is the pseudo-arc. Later on, several other known continua were constructed as topological realizations of topological graphs obtained as projective Fraïssé limits of appropriate classes of finite graphs with epimorphisms. Examples include the Lelek fan (Bartošová and Kwiatkowska) and the Menger curve (Panagiotopoulos and Solecki).

We show that finite connected graphs with confluent epimorphism form a projective Fraïssé class and we investigate the continuum obtained as the topological realization of its projective Fraïssé limit. We show that this continuum is indecomposable, but not hereditarily indecomposable, as arc-components are dense. It is one-dimensional, pointwise self-homeomorphic, but not homogeneous, and each point is the top of the Cantor fan. Moreover, it is hereditarily unicoherent, in particular, it does not embed a circle; however, it embeds a solenoid and the pseudo-arc.

This is joint work with W. J. Charatonik and R. P. Roe.

Catching sequences with ideals

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We review the most recent results that use the Invariant Ideal Axiom introduced by the authors to present a full (consistent) topological classification of countable sequential groups. We then show that IIA has sufficient power to be an effective tool in studying sequences in general countable (not necessarily sequential) groups and uncountable groups. As an application we answer some questions asked by D. Shakhmatov, A. Arkhangel'skii, and M. Tkachenko. We conclude by stating a number of open questions.

[1]The research of Michael Hrušák was supported by PAPIIT grants IN100317 and IN104220, and CONACyT grant A1-S-16164

Selective properties of products of Fréchet–Urysohn spaces

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We shall discuss properties of products of countable spaces involving diagonalizations of sequences of their dense subsets. Recall that a space X is *M-separable* if for every sequence $\langle D_n : n \in \omega \rangle$ of dense subsets of X there exists a sequence $\langle F_n : n \in \omega \rangle$ such that $F_n \in [D_n]^{<\omega}$ for all n and $\bigcup_{n \in \omega} F_n$ is dense in X. As it was shown by D. Barman and A. Dow in their papers published in 2011 and 2012, CH implies the existence of two countable Fréchet-Urysohn spaces with non-M-separable product, while PFA implies that all such products are M-separable. The talk will be among others devoted to the following

Theorem. *The existence of two Fréchet–Urysohn spaces with non-M-separable product is consistent with the MA.*

The proof relies on special mad families used to control convergent sequences. Such mad families do not exist under PFA but might exist in models of MA, as shown by A. Dow and S. Shelah in 2012.

The talk is based on a joint work in progress with *Serhii Bardyla* (University of Vienna, Austria) and *Fortunato Maesano* (University of Messina, Italy).

[1]The presenting author would like to thank the Austrian Science Fund FWF (Grant I 3709-N35) for generous support for this research.

CONTRIBUTED TALKS

The program of Toposym 2022 contains about 60 contributed talks. All invited talks are 25 minutes long, most of them presented in 4 parallel sessions. Slides for the talks are available on the conference website.

When is a quasi-uniformly continuous real-valued function on a quasi-uniform space bounded?

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It is well known that all quasi-uniformly continuous real-valued functions on a totally bounded quasi-uniform space are bounded. Note however that there exist quasi-uniform spaces with this property that are not totally bounded, that is, a quasi-uniform space with no unbounded quasi-uniformly continuous functions need not be totally bounded. In this talk, we give a necessary and sufficient condition that will allow all quasi-uniformly continuous real-valued functions on a quasi-uniform space to be bounded. We conclude the talk by giving analogous results for quasi-proximity spaces.

[1]The first author was partially supported by the NRF of South Africa

On the category of probabilistic topological convergence groups

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Generalizing the idea of Lowen – approach spaces [4], Jäger introduced the notion of probabilistic approach spaces [3]. Ahsanullah and Jäger presented a category of probabilistic convergence groups, studied uniformizability and metrization of probabilistic convergence groups, exploring some natural connections between the categories of probabilistic metric groups and probabilistic convergence groups [1, 2]. In this talk, we first show that the category of probabilistic topological convergence groups is *isomorphic* to the category of probabilistic approach groups under so-called triangle function $\tau\colon \Delta^+ \times \Delta^+ \longrightarrow \Delta^+$, where Δ^+ is the set of all *distance distribution functions* that plays a central role for probabilistic metric spaces. Furthermore, if we allow this triangle function τ to be *sup-continuous*, then among others, one can show that the category of symmetric probabilistic quasi-metric groups can be embedded into the category of probabilistic approach groups as a bircoreflective subcategory. Finally, we discuss the relationship between the categories of probabilistic topological convergence transformation groups and probabilistic approach transformation groups under the triangle function τ.

[1] T. M. G. Ahsanullah and G. Jäger, *Probabilistic uniformization and probabilistic metrization of probabilistic convergence groups*, Math. Slovaca, 67 (2017), pp. 985–1000.

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On completeness and topologizability of countable semigroups

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In this talk we discuss a connection between categorical closedness and topologizability of semigroups. For a class $\mathsf{T_1S}$ of T_1 topological semigroups we show that a countable semigroup X with finite-to-one shifts is injectively $\mathsf{T_1S}$-closed if and only if X is $\mathsf{T_1S}$-nontopologizable in the sense that every T_1 semigroup topology on X is discrete. Moreover, a countable cancellative semigroup X is absolutely $\mathsf{T_1S}$-closed if and only if every homomorphic image of X is $\mathsf{T_1S}$-nontopologizable. Also, we discuss a notion of a polybounded semigroup. It is proved that a countable semigroup X with finite-to-one shifts is polybounded if and only if X is $\mathsf{T_1S}$-closed if and only if X is $\mathsf{T_zS}$-closed, where $\mathsf{T_zS}$ is a class of Tychonoff zero-dimensional topological semigroups. We show that polyboundedness provides an automatic continuity of the inversion in T_1 paratopological groups and prove that every cancellative polybounded semigroup is a group.

[1]The presenting author is supported by FWF grant M 2967.

Hereditarily indecomposable continua as Fraïssé limits

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In 2006, Irwin and Solecki introduced projective Fraïssé theory of topological structures and showed that a pre-space of the pseudo-arc is the Fraïssé limit of the class of all finite linear graphs and quotient maps. They also characterized the pseudo-arc as the unique arc-like continuum $\mathbb{P}$ such that for every arc-like continuum Y, every $\varepsilon > 0$, and every continuous surjections $f, g \colon \mathbb{P} \to Y$ there is a homeomorphism $h \colon \mathbb{P} \to \mathbb{P}$ such that $\sup_{x \in \mathbb{P}} d(f(x), g(h(x))) < \varepsilon$.

We consider an approximate framework for Fraïssé theory where the pseudo-arc itself is the Fraïssé limit of the category $\mathcal{I}$ of all continuous surjections of the unit interval, in the category $\sigma\mathcal{I}$ of all arc-like continua and all continuous surjections. The characterizing condition above becomes the *projective homogeneity* condition in our framework.

Similarly, we may consider the category $\mathcal{S}$ of all continuous surjections of the unit circle, and the category $\sigma\mathcal{S}$ of all circle-like continua and all continuous surjections. It turns out there is no Fraïssé limit of $\mathcal{S}$ in $\sigma\mathcal{S}$. However, if we restrict to the subcategory $\mathcal{S}_P \subseteq \mathcal{S}$ of the maps whose degree uses only primes from a fixed set P, and the subcategory $\sigma\mathcal{S}_P \subseteq \sigma\mathcal{S}$ of circle-like continua that are limits of inverse sequences of $\mathcal{S}_P$-maps, with maps that can be approximated by $\mathcal{S}_P$-maps as morphisms, then the corresponding Fraïssé limit is the P-adic pseudo-solenoid $\mathbb{P}_P$, and it is characterized as the unique $\sigma\mathcal{S}_P$-object that is *projectively homogeneous*, or equivalently has the *projective extension property.*

[1]The authors were supported by GA ČR (Czech Science Foundation) grant EXPRO 20-31529X and RVO: 67985840.

Scattered P-spaces of weight ω_1

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Following M. Fréchet, if a space X can be embedded into a space Y, then we write $X \subset_h Y$. If $X \subset_h Y$ and $Y \subset_h X$, then we say that X and Y have the same *topological rank* (K. Kuratowski) or *dimensional type* (W. Sierpiński). Such relations are used in, for example, considering topological inclusion relation ([1], [2]). It is also a problem interesting in itself, as can be seen from [4], where the author has described the ordering $\subset_h$ in the class of all separable and scattered metric spaces. We consider the ordering $\subset_h$ in the class of all scattered P-spaces of weight ω_1, where a P*-space* is a regular space such that its G_δ sets are open. Moreover, in this case it suffices to examine subspaces of the ordinal number ω_2, as every scattered P-space of weight ω_1 is homeomorphic to such a subset.

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Some formulas for uniform spaces

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The report will present formulas for permutation of the absolutes of uniform spaces with extensions of Samuel, Hewitt and completions of uniform spaces.

Let (X, U) an arbitrary uniform space and U a uniformity on X generating the topology of the space X. We denote by $(\dot{X}, \dot{U})$ the absolute of the uniform space (X, U) [3], where $\dot{U}$ is the absolute of the uniform space U, $\dot{X}$ is the absolute of the topological space X in the sense of V. I. Ponamarev [1]. Let (sX, sU) be a Samuel extension [2], $(\nu X, \nu U)$ a Hewitt extension and $(\tilde{X}, \tilde{U})$ a completion of a uniform space (X, U) [3]. Then the following formulas are true:

(1) $(s\dot{X}, s\dot{U}) \simeq ((sX)^{\cdot}, (sU)^{\cdot})$;

(2) $(\tilde{\dot{X}}, \tilde{\dot{U}}) \simeq ((\tilde{X})^{\cdot}, (\tilde{U})^{\cdot})$;

(3) $(\nu\dot{X}, \nu\dot{U}) \simeq ((\nu X)^{\cdot}, (\nu U)^{\cdot})$.

[1] V. I. Ponomarev, *On spaces co-absolute with metric spaces*, Uspekhi matem. nauk 11 (1966), (in Russian).

[2] R. Engelking, *General topology*, Moscow, 1986 (in Russian).

[3] A. A. Borubaev, *Uniform topology and its applications*, Bishkek, 2021.

A proof of the Tree Alternative Conjecture for the Topological Minor Relation

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The Tree Alternative Conjecture states that the equivalence class of any tree (rooted or unrooted) under mutual embeddability is either 1 or infinite. We prove the analogous for the topological minor relation.

Theorem. *For any tree T*

(1) $|T| = 1$ *or* $|T| \geq \aleph_0$, *and*

(2) *for any* $r \in V(T)$, $|(T,r)| = 1$ *or* $|(T,r)| \geq \aleph_0$.

The above is proved by means of stratifying all trees into two complementary categories: those containing all *large* and those containing *small* trees. We then establish the following

Theorem. *For any large tree T,*

(1) $|T| \geq 2^{\aleph_0}$ *and*

(2) *for any* $r \in V(T)$, $|(T,r)| \geq 2^{\aleph_0}$.

Theorem. *For any small tree T,*

(1) $|T| = 1$ *or* $|T| \geq \aleph_0$, *and*

(2) *for any* $r \in V(T)$, $|(T,r)| = 1$ *or* $|(T,r)| \geq \aleph_0$.

On the existence of isovariant extensors for actions of locally compact groups

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The notion of an isovariant absolute extensor coincides, in fact, with the concept of the universal G-space in the sense of R.S. Palais used for the the classification of G-spaces. The existence of isovariant absolute extensors proved by Palais for a relatively restricted class of G-spaces (in particular, the G-spaces of this class are finite-dimensional of finite orbit type and the group G is assumed to be a compact Lie group) is not a trivial fact.

For a given locally compact group G, we consider the class $G\text{-}\mathcal{PM}$ of metrizable proper G-spaces with metrizable orbit spaces. Our main result is the following:

Theorem. *For every locally compact group G there exists a $G\text{-}\mathcal{PM}$-space which is an isovariant absolute extensor for the class $G\text{-}\mathcal{PM}$.*

As an immediate application of the main result we obtain the following corollary:

Theorem. *For every locally compact metrizable group G there exists a proper free action of the group G on the Hilbert space which makes it into an equivariant absolute extensor for free $G\text{-}\mathcal{PM}$-spaces.*

On the cardinality of a power homogeneous compactum

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In 2006 de la Vega showed that the cardinality of a homogeneous compactum X is at most $2^{t(X)}$. This was followed in 2007 by Arhangel'skii, van Mill, and Ridderbos who showed the same cardinality bound holds if X is a power homogeneous compactum. In this talk we show that the tightness $t(X)$ can be replaced with $at(X)\pi\chi(X)$, where the almost tightness $at(X)$ satisfies the property $wt(X) \leq at(X) \leq t(X)$. As $\pi\chi(X) \leq t(X)$ for a compactum X and $at(X) \leq t(X)$ for any space, this gives a formal improvement of the result of Arhangel'skii, van Mill, and Ridderbos.

Power homogeneity is used through the following key result. Note a set G is a G^c_κ-set of a space Y if there exists a family of open sets $\mathcal{U}$ in Y such that $|\mathcal{U}| \leq \kappa$ and $G = \bigcap \mathcal{U} = \bigcap_{U\in\mathcal{U}} \overline{U}$.

Let X be a power homogeneous Hausdorff space where $\pi\chi(X) \leq \kappa$. Suppose there exists a nonempty G^c_κ-set G and a set $H \in [X]^{\leq\kappa}$ such that $G \subseteq \overline{H}$. Then there exists a cover $\mathcal{G}$ of X consisting of G^c_κ-sets such that for all $G \in \mathcal{G}$ there exists $H_G \in [X]^{\leq\kappa}$ such that $G \subseteq \overline{H_G}$.

Another component of the proof of the main cardinality bound involves the notion of a T-free sequence, a stronger type of free sequence. We show that a compact subset of a space X contains no T-free sequence of length κ^+ when $\kappa = at(X)$. Further components involve results concerning the weak tightness $wt(X)$ and a cardinality bound for power homogeneous spaces due to Ridderbos.

Generalized Ważewski dendrites as projective Fraïssé limits

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In a recent preprint Włodzimierz J. Charatonik and Robert P. Roe investigated projective Fraïssé limits of trees with natural classes of maps inspired from continuum theory. Among other results they proved that the class of finite trees with monotone epimorphisms is a projective Fraïssé class and its projective Fraïssé limit is a prespace whose topological realization is the Ważewski dendrite W_3.

We introduce new types of maps between finite trees, called (weakly) coherent, and use them to realize many other generalized Ważewski dendrites as the topological realization of projective Fraïssé limits. In particular for all $P \subseteq \omega$ and all coinfinite $P \subseteq \omega + 1$ we construct a projective Fraïssé class whose limit has as topological realization the generalized Ważewski dendrite W_P.

By moving to the more general setting of Fraïssé categories and projection-embedding pairs developed by Wiesław Kubiś, we remove the coinfiniteness assumption on P and realize all generalized Ważewski dendrites as the topological realization of projective Fraïssé limits.

Characterization of (semi-)Eberlein compacta using retractional skeletons

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We provide a new characterization of (semi-)Eberlein compacta using the notion of a retractional skeleton. Using this characterization we give a new proof of the fact that continuous image of Eberlein compact is Eberlein and, generalizing this idea, we also obtain new structural results concerning continuous images of semi-Eberlein compacta in particular solving in positive a question asked by Kubis and Leiderman.

This is a joint work with C. Correa and J. Somaglia.

Generating subgroups of the circle using density functions

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This talk is based on the works done along with my students K. Bose, A. Ghosh and D. Dikranjan. In [3] a new version of characterized subgroups of the circle group $\mathbb{T}$ were introduced called "s-characterized subgroups" which are essentially different and strictly larger in size than the much investigated class of characterized subgroups, having cardinality $\mathfrak{c}$ but remaining nontrivial. Recently the notion has further been extended in [2] using the generalized version d_g^f of the natural density function introduced in [1] where $g : \mathbb{N} \to [0, \infty)$ satisfies $g(n) \to \infty$ and $\frac{n}{g(n)} \nrightarrow 0$ whereas f is an unbounded modulus functions. These subgroups have the same feature as the s-characterized subgroups [3]. But at the same time the utility of this more general approach is justified by constructing new and nontrivial subgroups for suitable choice of f and g.

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[2] P. Das and A. Ghosh, *Generating subgroups of the circle using a generalized class of density functions*, Indag. Math., 32 (2021), pp. 598–618.

[3] D. Dikranjan, P. Das, and K. Bose, *Statistically characterized subgroups of the circle*, Fund. Math., 249 (2020), pp. 185–209.

Embedding of the Higson compactification into the product of adelic solenoids

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The Higson compactification $\bar{X}$ of X is defined by means of bounded slowly oscillating continuous functions $f : X \to \mathbb{R}$. If C_h is the set of all such functions, then $\bar{X}$ is homeomorphic to the closure of X under the embedding

$$(f)_{f\in C_h} : X \to \prod_{f\in C_h} [\inf f, \sup f].$$

Theorem. *Every simply connected proper geodesic metric space X admits an embedding of its Higson compactification into the product of adelic solenoids*

$$F : \bar{X} \to \prod_{\mathcal{A}} \Sigma_\infty$$

that induces an isomorphism of 1-dimensional Čech cohomology.

As a corollary we obtain the following

Theorem. *For any p and any simply connected finite dimensional proper geodesic metric space X its Higson compactification can be essentially embedded into the product of Knaster continua K_p.*

We recall that the Knaster continuum $K_p = \Sigma_p/\sim$ is the quotient space under the identification $x \sim -x$.

[1]The first author was partially supported by Simons Foundation.

Planar absolute retracts and countable structures

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A compact space is said to be an absolute retract (AR) if it is homeomorphic to a retract of the Hilbert cube. In particular, every AR is a locally connected, simply connected continuum. The following lemma shows an interesting property of ARs in the plane.

Lemma. *If $X, Y \subseteq \mathbb{R}^2$ are absolute retracts, then X is homeomorphic to Y if and only if ∂X is homeomorphic to ∂Y.*

A sketch of the proof of this lemma is going to be shown, as well as examples witnessing the importance of the assumption that X and Y are ARs.

This lemma has a nice consequence belonging to the field of invariant descriptive set theory (a discipline studying complexities of equivalence relations on standard Borel spaces).

Theorem. *The homeomorphism equivalence relation on the class of planar absolute retracts is classifiable by countable structures.*

On the other hand, we also have the following theorem.

Theorem. *The homeomorphism equivalence relation on the class of absolute retracts contained in $\mathbb{R}^3$ is not classifiable by countable structures.*

Question. Is the homeomorphism equivalence relation on the class of planar absolute neighborhood retracts classifiable by countable structures?

Separable reductions, rich families and projectional skeletons in non-separable Banach spaces

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We show that continuity, differentiablity and the like are separably reducible properties. In this course, we meet the concepts of cofinal and reach families of separable subspaces of a Banach space. The rich families fit very well for indexing projectional skeletons (PS), this modern substitute of PRI introduced by W. Kubiś. The PS are then used for study of various classes of nonseparable Banach spaces like Asplund spaces, WCG spaces and many relatives of them.

Box and nabla products that are D-spaces

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It is a well known open problem whether paracompact spaces are D. The even more famous box product problem, on the other hand, is concerned with when box or nabla products are paracompact. We will present results showing that many box and nabla products are D-spaces, indeed quite often hereditarily D. This will include most cases where it is known the box or nabla product is paracompact.

Here a space X is D if for every assignment, U, of an open neighborhood to each point x in X there is a closed discrete E such that $\bigcup\{U(x) : x \in E\} = X$. The box product, $\square X^\omega$, is X^ω with topology generated by all $\prod_n U_n$, where every U_n is open. The nabla product, ∇X^ω, is obtained from $\square X^\omega$ by quotienting out mod-finite.

Universal flows revisited

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Let G be a topological group. A compact space X with a continuous G-action is a *G-flow*. Let $\mathcal{C}$ be a class of G-flows. $X \in \mathcal{C}$ is *universal* in $\mathcal{C}$ if X maps onto any other member of $\mathcal{C}$ by a continuous G-equivariant map. We discuss the existence of universal metric G-flows for countable discrete groups G.

On some relative versions of Menger and Hurewicz properties

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Let $\mathcal{U}$ be a cover of a space X and A be a subset of X; the star of A with respect to $\mathcal{U}$ is the set $st(A,\mathcal{U}) = \bigcup\{U : U \in \mathcal{U} \text{ and } U \cap A \neq \emptyset\}$. In this talk we consider some recent relative star versions of Menger property and the corresponding Hurewicz-type properties, introduced in [1]. We show that the considered properties are between countable compactness and the property of having countable extent and study the behavior with respect the product with a compact space. Among other things we answer to some recent questions posed in [1].

[1] L. Kočinac, S. Konca, and S. Singh, *Set star-menger and set strongly star-menger spaces*, Math. Slovaka, 72 (2022), pp. 185–196.

[1]The author was partially supported by the "National Group for Algebraic and Geometric Structures, and their Applications" (GNSAGA – INdAM).

On some results about cardinal inequalities for topological spaces

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Two of the most celebrated cardinal inequalities, which are valid for every Hausdorff topological space X, are Arhangel'skiĭ's inequality $|X| \leq 2^{\chi(X)L(X)}$, and Hajnal–Juhász' inequality $|X| \leq 2^{\chi(X)c(X)}$. The two inequalities are important, in particular, because they show that the two pairs of cardinal functions $L(X)$ and $\chi(X)$, and $c(X)$ and $\chi(X)$, respectively, are sufficient to give an upper bound for the cardinality of a Hausdorff topological space. But even Pospíšil's inequality (from 1937) $|X| \leq d(X)^{\chi(X)}$, which is also valid for every Hausdorff space X, gives always the same or a lower upper bound for the cardinality of X than the above two inequalities. This fact explains why there are so many improvements in the literature of these two inequalities.

In this talk we will compare some known results about cardinal inequalities for topological spaces and we will mention some new improvements (of some) of the above inequalities. In particular, we will mention Gotchev–Tkachenko–Tkachuk's inequality from 2016 that $|X| \leq \pi w(X)^{\mathrm{ot(X)}\psi_{\mathrm{c}}(\mathrm{X})}$, where ot(X) is the o-tightness of X. This inequality is valid for every Hausdorff space X and it improves not only Hajnal–Juhász' inequality but also Sun's inequality $|X| \leq \pi\chi(X)^{c(X)\psi_c(X)}$, where $\pi\chi(X)$ is the π-character of X.

We will finish with our recent result that $|X| \leq \pi w(X)^{\mathrm{dot(X)}\cdot\psi_{\mathrm{c}}(\mathrm{X})}$, where $\pi w(X)$ is the π-weight and dot(X) is the dense o-tightness of X, which improves all of the above-mentioned cardinal inequalities.

Closure spaces, countable conditions and the axiom of choice

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A closure space or a Čech closure space is a pair (X, c) where X is a set and $c : 2^X \to 2^X$ is a closure operator such that:

(i) $c(\emptyset) = \emptyset$;

(ii) $A \subseteq c(A)$;

(iii) $c(A \cup B) = c(A) \cup c(B)$.

In other words, a Čech closure is a topological (or Kuratowski) closure where the idempotency of the closure is not imposed.

In this talk we will discuss how to transpose to closure spaces some countable notions usual in topological spaces such as: separability, Lindelöfness, first and second countability, ... and study how they compare to each other using the axiom of choice, some weak forms of choice or in a choice-free context.

[1]The author was partially funded by the Portuguese Government through FCT/MCTES - UIDB/00324/2020.

A Banach space C(K) reading the dimension of K

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In 2004 Koszmider constructed a compact Hausdorff space K such that whenever L is compact Hausdorff and the Banach spaces of continuous functions $C(K)$ and $C(L)$ are isomorphic, L is not zero-dimensional. We show that, assuming Jensen's diamond principle ($\diamondsuit$), the following strengthening of the above result holds:

Theorem. *Assume* $\diamondsuit$*. Let* $n \in \mathbb{N}$*. There is a compact Hausdorff space* K*, such that if* L *is compact Hausdorff and* $C(K) \sim C(L)$*, then the covering dimension of* L *is equal to* n*.*

The constructed space is a modification of Koszmider's example. It is a separable connected compact space with the property that every linear bounded operator $T : C(K) \to C(K)$ is a weak multiplication i.e. it is of the form $T(f) = gf + S(f)$, where $g \in C(K)$ and S is a weakly compact operator on $C(K)$.

[1] The author was supported by the NCN (National Science Centre, Poland) research grant no. 2021/41/N/ST1/03682

Digital-topological k-group structures on digital objects

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Motivated by the typical topological group, we have recently developed the notion of a digital-topological k-group (DT-k-group for brevity) derived from a digital object $X(\subset \mathbb{Z}^n)$ with digital k-connectivity, i.e., (X, k). In relation to this work, we need the most suitable adjacency relation in a digital product $X \times X$ such as a G_{k^*}-adjacency relation which can support the G_{k^*}-connectedness of the digital space $(X \times X, G_{k^*})$, (G_{k^*}, k)-continuity of the map $\alpha : (X \times X, G_{k^*}) \to (X, k)$, and k-continuity of the inverse map $\beta : (X, k) \to (X, k)$.

We prove that $(\mathbb{Z}^n, k, +, \cdot)$ is an infinite DT-k-group and $(SC_k^{n,l}, *, \star)$ is a finite DT-k-group, where the operations $*$ and $\star$ are particularly defined.

Given two DT-k_i-groups $(X_i, k_i, *_i, \star_i)$, $i \in \{1, 2\}$, assume the digital product $X_1 \times X_2$. Then we can raise a query. Under what k-adjacency of $X_1 \times X_2$ do we have the product property of the given two DT-k_i-groups $(X_i, k_i, *_i, \star_i)$, $i \in \{1, 2\}$?

Finally, we will suggest some applicable areas in the fields of applied mathematics and computer science.

[1]The first author was partially supported by NRF (National Research Fund of Korea).

Ramsey theorem for trees with successor operation

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We discuss a new Ramsey-type theorem for trees. On regularly branching trees it can be seen as a common generalization of the Milliken tree theorem and the Carlson–Simpson theorem. The main new concept is the use of sucessor operation enabling us to work with trees with unbounded branching and identify subtrees that are isomorphic to the original tree. Using this result we can give new direct proofs of recent results by Dobrinen [3, 2], Balko, Chodounský, Hubička, Konečný, Vena [1] and Zucker [4] as well as identify new structures with finite big Ramsey degrees.

This is joint work with Balko, Chodounský, Dobrinen, Konečný, Nešetřil, Vena and Zucker.

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[2] N. Dobrinen, *The Ramsey theory of Henson graphs*, submitted, arXiv:1901.06660, 2019.

[3] N. Dobrinen, *The Ramsey theory of the universal homogeneous triangle-free graph*, Journal of Mathematical Logic, 20 (2020), p. 2050012.

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[1]Supported by the project 21-10775S of the Czech Science Foundation (GAČR)

Periodicity of solenoidal automorphisms

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Characterization of the sets of periodic points of a family of dynamical systems is a well studied problem in the literature. Here, we consider this problem for the family of automorphisms on a solenoid. By definition, a solenoid Σ is a compact connected finite dimensional abelian group. Equivalently, a topological group Σ will be a solenoid if its Pontryagin dual $\widehat{\Sigma}$ is a subgroup of $\mathbb{Q}^n$ and also contains $\mathbb{Z}^n$ as a subgroup for some positive integer n. When the dual is equal to $\mathbb{Z}^n$, the solenoid is actually an n-dimensional torus, while its known as a full solenoid when the dual is $\mathbb{Q}^n$. Previously the characterization has been done on n-dimensional torus, full solenoids and also for the alternate description by considering a one-dimensional solenoid as the inverse limit of a sequence of maps on unit circle.

This talk is based upon a pre-print, related to our recent work about the extension of periodic point characterization to n-dimensional solenoids, whose duals are subgroups of algebraic number fields. Here, we used the theory of adeles for describing a solenoid and the periodic points of its automorphisms. The ring of adeles $\mathbb{A}_{\mathbb{K}}$ of an algebraic number field $\mathbb{K}$, is the restricted product of $\mathbb{K}_v$'s with respect to $\Re_v$'s, where $\mathbb{K}_v$ is the completion of $\mathbb{K}$ with respect to a place v and $\Re_v$ is an open, unique maximal compact subring of $\mathbb{K}_v$.

[1]Both the authors thank SERB-DST, Govt. of India for financial support.

On σ-metacompact function spaces

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We introduce the following property of a family $\mathcal{L}$ of subsets of a set S:

($*$) For all $N \in \mathcal{L}$ and $x \in S$, there exists a finite subset A of N such that, for each $L \in \mathcal{L}$, if $x \in L$ and $L \cap N \neq \emptyset$, then $L \cap A \neq \emptyset$.

We consider compact spaces which have a k-network with property ($*$). Examples of spaces which do not admit a k-network with ($*$) include $\beta\omega$, a compact scattered space of height $\omega + 1$ and the one-point compactification of a tree-space.

Theorem. *If K is a compact space which has a k-network with property ($*$), then $C_p(K)$ is hereditarily σ-metacompact.*

Supercompact spaces are usually defined by the existence of a "binary" subbase for the closed subsets, but according to a known and easy result, every supercompact space has a binary closed k-network.

Proposition. *A family $\mathcal{L}$ of compact closed subsets of a space X is binary if, and only if, for all $N \in \mathcal{L}$ and $x \in X$, there exists $a \in N$ such that, for each $L \in \mathcal{L}$, if $x \in L$ and $L \cap N \neq \emptyset$, then $a \in L$.*

Hence every supercompact space has a k-network with ($*$).

Corollary. *$C_p(K)$ is hereditarily σ-metacompact for every supercompact space K.*

Corollary. *$C_p(K)$ is hereditarily σ-metacompact for every dyadic space K.*

Topologies related to (I)-envelopes

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The (I)-envelope of a set A in a dual Banach space is defined by

$$(\mathrm{I})\text{-env}(A) = \bigcap \left\{ \overline{\bigcup_{n\in\mathbb{N}} \overline{\operatorname{conv} A_n}^{w^*}}^{\|\cdot\|} : A = \bigcup_{n\in\mathbb{N}} A_n \right\}.$$

This notion, inspired by the notion of (I)-generation from [2], was introduced in [3]. It was used in [4, 1, 5], in particular to characterize Grothendieck property and its quantitative version. (I)-env(A) is a norm-closed convex set and $\overline{\operatorname{conv} A}^{\|\cdot\|} \subset (\mathrm{I})\text{-env}(A) \subset \overline{\operatorname{conv} A}^{w^*}$ for any set A. We will address the following natural problem:

Question. Let X be a Banach space. Is there a (locally convex) topology τ on X^* such that $(\mathrm{I})\text{-env}(A) = \overline{\operatorname{conv} A}^{\tau}$ for each $A \subset X^*$?

The answer to the 'locally convex' version is 'sometimes yes, sometimes no', but a complete characterization is still missing. The 'topological' version is widely open and is connected to several interesting intermediate topologies on X^*.

[1] H. Bendová, *Quantitative Grothendieck property*, J. Math. Anal. Appl., 412 (2014), pp. 1097–1104.

[2] V. P. Fonf and J. Lindenstrauss, *Boundaries and generation of convex sets*, Israel J. Math., 136 (2003), pp. 157–172.

[3] O. F. K. Kalenda, *(I)-envelopes of closed convex sets in Banach spaces*, Israel J. Math., 162 (2007), pp. 157–181.

[4] O. F. K. Kalenda, *(I)-envelopes of unit balls and James' characterization of reflexivity*, Studia Math., 182 (2007), pp. 29–40.

[5] J. Lechner, *1-Grothendieck $C(K)$ spaces*, J. Math. Anal. Appl., 446 (2017), pp. 1362–1371.

Selections principles in uniform topology

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Z. Frolik posed the following general problem: To find and to investigate uniform analogues of the most important classes of topological spaces and continuous mappings (the “uniformization” problem) at a seminar on topology at the Charles University (Prague). Therefore, the problem of “uniformization” of the topological spaces theory is relevant.

Selection principles of the theory of uniform spaces was first studied by L. Kocinac [1]. It follows from the definitions of uniform Menger, uniform Hurewicz, and uniform Rothberger spaces that they are intermediate between precompact and pre-Lindelöf spaces, and therefore should have many good properties.

In this work uniform Menger, uniform Hurewicz and uniform Rothberger spaces are studied. In particular, these uniform properties extend to uniformly continuous mappings.

[1] L. Kocinac, *Selection principles in uniform spaces*, Note di Matematica, 22 (2003), pp. 127–139.

On μ-completeness of uniform spaces and uniformly continuous mappings

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In this talk the μ-complete uniform spaces are studied, i.e. those spaces, where every Cauchy filter with base of cardinality $\leq \mu$ converges. We introduce a new concept of index of μ-completeness denoted by $ic_\mu(X,U)$ of a uniform space (X,U) and the Dieudonne μ-complete space X, and also index of μ-completeness $ic_\mu(f)$ of the uniform continuous mapping $f:(X,U)\to(Y,V)$ between uniform spaces (X,U) and (Y,V).

Some characteristics of these concepts are established.

(1) $ic_\mu(X,U)=1$ iff (X,U) is uniformly locally μ-compact space;

(2) $ic_\mu(f)=1$ iff f is uniformly locally μ-quasi-perfect mapping;

(3) Tychonoff space (X,U) is Dieudonne μ-complete iff a uniform space (X,U_X) with a universal uniformity U_X is μ-complete.

On a generalization of the selection theorem for $\mathcal{C}$-spaces

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We obtain a new proof of the Uspenskij selection theorem for $\mathcal{C}$-spaces. The new proof based on Ostrand's colored dimension theorem and uses notion of striped multivalued selections. This result allows us to generalize the Uspenskij theorem to stratified $\mathcal{C}$-spaces.

Definition. A multivalued map $G\colon X \rightsquigarrow Y$ is called *$\mathcal{U}$-continuous* if $\{x \in X \mid L \subset G(x)\}$ is open for all compact $L \subset Y$.

Theorem. *Let X be a paracompact Hausdorff $\mathcal{C}$-space with closed n-dimensional subspace $X_0 \subset X$, and let Y be a Hausdorff space. Suppose $X \setminus X_0$ is a paracompact $\mathcal{C}$-space, and $G\colon X \rightsquigarrow Y$ is a $\mathcal{U}$-continuous multivalued map. If $G(x) \in C^{n-1}$ for all $x \in X_0$, and $G(x) \in C^{\infty}$ for all $x \in X \setminus X_0$, then there exists a selection $s\colon X \to Y$ of map G.*

Suppose $X_0 = \varnothing$, then we get classical Uspenskij theorem.

Tame locally convex spaces

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Motivated by Rosenthal's famous l^1-dichotomy in Banach spaces, Haydon's theorem, and additionally by recent works on tame dynamical systems, we introduce the class of *tame* locally convex spaces. This is a natural locally convex analogue of *Rosenthal* Banach spaces (for which any bounded sequence contains a weak Cauchy subsequence). Our approach is based on a bornology of *tame* subsets which in turn is closely related to eventual fragmentability, and some purely topological ideas. This leads, among others, to a generalization of Haydon's theorem for locally convex spaces, a version of Rosenthal's dichotomy strengthening a result of W.M. Ruess, and an extension of the Davis–Figiel–Johnson–Pelczyński (DFJP) factorization technique. It also relates to recent papers by A. Leiderman and V. Uspenski [3], and S. Gabriyelyan and T. Banakh [1]. This project is based on a submitted joint work [2] with M. Megrelishvili.

[1] T. Banakh and S. Gabriyelyan, *Free locally convex spaces.* To appear in FILOMAT.

[2] M. Komisarchik and M. Megrelishvili, *Tameness and rosenthal type locally convex spaces*, 2022. arXiv:2203.02368.

[3] A. Leiderman and V. Uspenskij, *Is the free locally convex space $L(X)$ nuclear?*, 2021. arXiv:2106.13413.

Big Ramsey degrees and infinite languages

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Study of big Ramsey degrees is an infinitary extension of the study of Ramsey classes. Like the Kechris–Pestov–Todorcevic correspondence [1] gives a link between Ramsey classes and topological dynamics, big Ramsey degrees are also closely connected to dynamical properties of the automorphism groups of homogeneous structures.

The area of big Ramsey degrees is currently seeing fast development. Recently, we were able to show that an unconstrained homogeneous relational structure has finite big Ramsey degrees if and only if it is ω-categorical (if and only if its *tree of n-types* is finitely branching for every n). In particular, this is the first time we were able to handle structures in infinite languages. This is done by applying the product Milliken theorem for arbitrarily large (finite) products of trees.

This is joint work with Samuel Braunfeld, David Chodounský, Noé de Rancourt, Jan Hubička and Jamal Kawach.

[1] A. S. Kechris, V. G. Pestov, and S. Todorcevic, *Fraïssé limits, Ramsey theory, and topological dynamics of automorphism groups*, Geom. Funct. Anal., 15 (2005), pp. 106–189.

Recognizing the topologies on subspaces of L^p-spaces on metric measure spaces

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As applications of the theory of infinite-dimensional manifolds, topological structures of function spaces have been researched, and “typical” convex sets in Hilbert space ℓ_2 have been recognized among them. For instance, the linear span ℓ_2^f of the orthonormal basis of ℓ_2 is detected in several function spaces as a factor. In this talk, we shall study the topological types with ℓ_2^f as a factor of subspaces of L^p-spaces on metric measure spaces.

Suppose that X is a Borel-regular Borel metric measure space such that every open ball has a positive and finite measure. For $1 \leq p < \infty$, the L^p-space $L^p(X)$ on X is homeomorphic to ℓ_2 when X is infinite and separable. We investigate the topology on the subspace consisting of uniformly continuous maps in $L^p(X)$.

Characterizing compact sets in function spaces plays important roles in the study on them. By using average functions, we shall give a generalization of the Kolmogorov–Riesz theorem, which is an L^p-version of the Ascoli–Arzelà one. Applying that criterion, we recognize the topology on the subspace consisting of lipschitz maps with bounded supports in $L^p(X)$.

Arbitrarily large countably compact free Abelian groups

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A result of A. Tomita shows that a nontrivial free Abelian group does not admit a group topology whose countable power is countably compact.

In our work, we show that if there are $\mathfrak{c}$ incomparable selective ultrafilters, then, for every infinite cardinal κ such that $\kappa^\omega = \kappa$, there exists a group topology on the free Abelian group of cardinality κ without nontrivial convergent sequences and such that every finite power is countably compact.

This answers a question of Dikranjan and Shakhmatov that was posed in a survey by Comfort, Hoffman and Remus.

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[2]The third author received financial support from FAPESP 2017/15502-2.
[3]The fourth author received financial support from FAPESP 2016/26216-8.

The Menger property is l-invariant

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For a Tychonoff space X, by $C_p(X)$ we denote the space of all continuous real-valued functions on X endowed with the topology of pointwise convergence. Recall that a space X has the *Menger property* if for every sequence $(\mathcal{U}_n)_{n\in\mathbb{N}}$ of open covers of X, there is a sequence $(\mathcal{V}_n)_{n\in\mathbb{N}}$ such that for every n, $\mathcal{V}_n$ is a finite subfamily of $\mathcal{U}_n$ and the family $\bigcup_{n\in\mathbb{N}} \mathcal{V}_n$ covers X.

An old question of A.V. Arhangel'skii asks if the Menger property of X is preserved by homeomorphisms of the space $C_p(X)$. A similar question can also be asked for linear homeomorphisms of $C_p(X)$-spaces. In 2020 M. Sakai gave the affirmative answer in the linear case under an additional assumption on X. We show that the answer in the linear case is affirmative in the full generality. Our method can also be applied to prove analogous theorems for other, related covering type properties.

Δ-spaces X and distinguished spaces $C_p(X)$

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Definition. (G. M. Reed, E. van Douwen) A subset of reals $X \subset \mathbb{R}$ is said to be a Δ-*set* if for every decreasing sequence $\{D_n : n \in \omega\}$ of subsets of X with empty intersection, there is a decreasing sequence $\{V_n : n \in \omega\}$ consisting of open subsets of X, also with empty intersection, and such that $D_n \subset V_n$ for every $n \in \omega$.

Definition. (A. Grothendieck) A locally convex space (lcs) E is called *distinguished* if the strong dual of E (i.e. the topological dual of E endowed with the strong topology) is barrelled.

Theorem. *Let X be a Tychonoff space. A lcs $C_p(X)$ is distinguished if and only if for each $f \in \mathbb{R}^X$ there is a bounded $B \subset C_p(X)$ such that f belongs to the closure of B in $\mathbb{R}^X$.*

We say that a Tychonoff space X is a Δ-*space* if X satisfies property Δ, as in the first Definition above.

Theorem. *Let X be a Tychonoff space. A lcs $C_p(X)$ is distinguished if and only if X is a Δ-space.*

My talk will be devoted to the main results about Δ-spaces which are published recently in several joint works.

Some compact-type and Lindelöf-type relative versions of star-covering properties

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Given a topological space X, a subset A and an open cover $\mathcal{U}$ of it, the star of A with respect to $\mathcal{U}$ is defined by the set $st(A,\mathcal{U}) = \bigcup\{U \in \mathcal{U} : U \cap A \neq \emptyset\}$. In last decades, many ways to cover a set with stars were discovered and studied. Recently, new classes of star covering properties, defined as relative versions of known ones, were introduced by Kočinac, Konca and Singh (see [1] and [2]). We study some of these compact-type and Lindelöf-type properties.

[1] L. Kočinac, S. Konca, and S. Singh, *Set star-menger and set strongly star-menger spaces*, Math. Slovaka, 72 (2022), pp. 185–196.

[2] L. Kočinac and S. Singh, *On the set version of selectively star-CCC spaces*, J. Math., (2020), pp. Article ID 9274503, 7 pages.

[1] The presenting author gracefully aknowledges the financial support of University of Palermo.

Descriptive complexity in number theory and dynamics

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Informally, a real number is *normal in base* b if in its b-ary expansion, all digits and blocks of digits occur as often as one would expect them to, uniformly at random. We will denote the set of numbers normal in base b by $N(b)$. Kechris asked several questions involving descriptive complexity of sets of normal numbers. The first of these was resolved in 1994 when Ki and Linton proved that $N(b)$ is Π^0_3-complete. Further questions were resolved by Becher, Heiber, and Slaman who showed that $\bigcap_{b=2}^{\infty} N(b)$ is Π^0_3-complete and that $\bigcup_{b=2}^{\infty} N(b)$ is Σ^0_4-complete. Many of the techniques used in these proofs can be used elsewhere. We will discuss recent results where similar techniques were applied to solve a problem of Sharkovsky and Sivak and a question of Kolyada, Misiurewicz, and Snoha. Furthermore, we will discuss a recent result where the set of numbers that are continued fraction normal, but not normal in any base b, was shown to be complete at the expected level of $D_2(\Pi^0_3)$. An immediate corollary is that this set is uncountable, a result (due to Vandehey) only known previously assuming the generalized Riemann hypothesis.

[1]The first author was supported by grant 2019/34/E/ST1/00082 for the project "Set theoretic methods in dynamics and number theory," NCN (The National Science Centre of Poland).

[2]The second author was supported by NSF grant DMS-1800323.

On zero-dimensional subspaces of Eberlein compacta and a characterization of ω-Corson compacta

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Recall that a compact space K is Eberlein compact if it can be embedded into some Banach space X equipped with the weak topology. The first part of our talk will be devoted to the known problem of the existence of nonmetrizable compact spaces without nonmetrizable zero-dimensional closed subspaces. Several such spaces were obtained using some additional set-theoretic assumptions. Recently, P. Koszmider constructed the first such example in ZFC. We investigate this problem for the class of Eberlein compact spaces. We construct such Eberlein compacta, assuming the existence of a Luzin set. We also show that it is consistent with ZFC that each Eberlein compact space of weight greater than ω_1 contains a nonmetrizable closed zero-dimensional subspace.

A compact space K is *ω-Corson compact* if, for some set Γ, K is homeomorphic to a subset of the σ-product of real lines

$$\sigma(\mathbb{R}^\Gamma) = \{x \in \mathbb{R}^\Gamma : |\{\gamma : x(\gamma) \neq 0\}| < \omega\}.$$

Clearly, every ω-Corson compact space is Eberlein compact. We will present a characterization of ω-Corson compact spaces obtained jointly with Grzegorz Plebanek and Krzysztof Zakrzewski.

Orderable groups and semigroup compactifications

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This project is dedicated to Eli Glasner on the occasion of his 75th birthday. Our aim is to find some new links between linear (circular) orderability of groups and topological dynamics. We suggest natural analogs of the concept of algebraic orderability for *topological* groups involving order-preserving actions on compact spaces and the corresponding enveloping semigroups in the sense of R. Ellis.

This approach leads to several natural questions. Some of them might be useful also for discrete (countable) orderable groups.

We study the following questions:

Question. Which topological groups can be embedded into the topological group $H_+(K)$ of all circular (linear) order-preserving homeomorphisms of K, endowed with compact-open topology, for some circularly (resp., linearly) ordered compact space K.

Question. Which topological groups G admit proper linearly (circularly) order compact right topological semigroup compactification $G \hookrightarrow S$? When such S is: a) metrizable? b) hereditarily separable? c) first countable?

[1]This research was supported by a grant of the Israel Science Foundation (ISF 1194/19)

On the additivity of strong homology for locally compact second countable spaces

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Recently Bergfalk and Lambie-Hanson showed that in the weakly compact Hechler model, the higher derived limits $\lim^n \mathbf{A}$ vanish for all n, where $\mathbf{A}$ is a certain inverse system of abelian groups indexed by ω^ω. We generalize this result and use it to show that in the weakly compact Hechler model, strong homology is additive for the class of locally compact second countable spaces.

[1]The first author's research and travel to attend TOPOSYM was supported by US NSF grant DMS-1854367

Some topological properties of the N_τ^φ-nucleus of a space X

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Let X be a T_1-space, φ be a cardinal-valued function, and τ be a cardinal number.

The N_τ^φ-nucleus of the space X is the space

$$N_\tau^\varphi X = \{\mathfrak{M} \in NX: \text{ there exists } F \in \mathfrak{M} \text{ such that } \varphi(F) \leq \tau\},$$

where, the set NX of all complete linked systems (CLS) of a space X.

Assume that $\mathfrak{M} \in N_\tau^\varphi$-basement of the CLS $\mathfrak{M}$ is the family

$$\mathfrak{F}_\tau^\varphi(\mathfrak{M}) = \{F \in \mathfrak{M}: \ \varphi(F) \leq \tau\}.$$

A topological space X is said to be N_τ^φ-nuclear if $N_\tau^\varphi X = NX$.

As φ, we take a density function d. Let $\tau = \omega$.

Theorem. *For every infinite compact space X the following conditions are equivalent:*

(1) *The space X satisfies the second axiom of countability;*

(2) *The space $N_\omega^d X$ satisfies the second axiom of countability;*

(3) *The space NX satisfies the second axiom of countability.*

Theorem. *For every infinite compact space X the following conditions are equivalent:*

(1) *The space X has the Souslin property;*

(2) *The space $N_\omega^d X$ has the Souslin property;*

(3) *The space NX has the Souslin property.*

The pseudo-means of the pseudo-arc

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A *continuum* is a compact connected metric space with more than one point. Let X be a continuum, a *pseudo-mean* for X is a continuous retraction $r : X \times X \to \Delta X$ where $\Delta X = \{(x, x) : x \in X\}$. Any continuum X admits two *trivial pseudo-means*, the first assings to each ordered pair (x, y) the pair (x, x) and the second the pair (y, y). Most of the known continua admit non-trivial pseudo-means. In 2017 Łysko conjectured that the pseudo-arc only admits the trivial pseudo-means.

We will talk about the pseudo-arc and how this is an example of a continuum that only admits the trivial pseudo-means, that is, the Łysko's conjecture is true.

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On uniform real complete spaces

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A uniform space (X, U) is called pre-real compact if its uniformity U is generated by all uniform continuous functions [1] [2].

Theorem. *For each uniform space (X, U) there is exactly one (up to an uniform homeomorphism) uniformly real complete space $(\theta_U X, \theta_U)$ with the following properties:*

(1) *There is an uniformly homeomorphic embedding $i : (X, U_F) \to (\theta_U X, \theta_U)$, for which $(\theta_U X, \theta_U)$ is the completion of the uniform space (X, U_F), where U_F is the maximal functional uniformity contained in U.*

(2) *For any continuous function $f : (X, U) \to (R, E_R)$, there is an uniformly continuous function $\tilde{f} : (\theta_U X, \theta_U) \to (R, E_R)$ such that $\tilde{f} \circ i = f$.*

Moreover, the spaces $(\theta_U X, \theta_U)$ also satisfy the condition:

(3) *For each uniformly continuous mapping $f : (X, U) \to (Y, \mathfrak{M})$ of the uniform space (X, U) into an arbitrary uniformly real complete space $(Y, \mathfrak{M})$, there is an uniform mapping $\tilde{f} : (\theta_U X, \theta_U) \to (Y, \mathfrak{M})$ such that $\tilde{f} \circ i = f$.*

[1] R. Engelking, *General topology*, Moscow, 1986 (in Russian).

[2] A. A. Borubaev, *Uniform topology and its applications*, Bishkek, 2021.

ZFC solution to 9 problems of Tkachuk on functional countability

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A Tychonoff space is called *functionally countable* if every real-valued continuous function has countable range. A compact space is functionally compact iff it is scattered. At this year's STDC conference, Vladimir Tkachuk posed 14 problems on functional countability. A ZFC solution is given for 9 of these problems, of which the most demanding is Question 8: If X is a compact, Fréchet–Urysohn space such that $X^2 \setminus \Delta_X$ is functionally countable, must X be separable? The counterexample has 2^ω isolated points and is the one-point compactification of a finer topology on $(0, 1)$ such that the non-isolated points form a closed discrete subspace.

On entropies in quasi-metric spaces

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Quasi-uniform entropy $h_{QU}(\psi)$ is defined for a uniformly continuous self-map ψ on a quasi-metric space (X, q). General statements are proved about this entropy, and it is shown that the quasi-uniform entropy $h_{QU}(\psi, q)$ is less or equals to the uniform entropy $h_U(\psi, q^s)$ for a uniformly continuous self-map ψ on a quasi-metric space (X, q). Finally we proved that the completion theorem for quasi-uniform entropy holds in the class of all join-compact quasi-metric spaces.

Continuity with or without ideal

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Triple $\langle X, \tau, \mathcal{I} \rangle$, where τ is a topology on the set X and $\mathcal{I}$ is an ideal on X is called *ideal topological space*. *Local function*, defined by

$$A^*_{(\tau,\mathcal{I})} = \{x \in X : A \cap U \notin \mathcal{I} \text{ for each } U \in \tau(x)\}$$

generates topology τ^* on X (which is finer than τ) in such way that F is closed in τ^* iff $F^* \subseteq F$.

If $f : \langle X, \tau_X \rangle \to \langle Y, \tau_Y \rangle$ is a continuous (open, closed) function, its "idealisation" does not have to be continuous (open, closed). We give several sufficient conditions when mapping remains continuous (open, closed) and through several examples we illustrate that the conditions we considered can not be weakened.

[1]This talk is supported by the Science Fund of the Republic of Serbia, Grant No. 7750027: Set-theoretic, model-theoretic and Ramsey-theoretic phenomena in mathematical structures: similarity and diversity – SMART

Minimal non-trivial closed hereditary coreflective subcategories in categories of topological spaces

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By **Top** we denote the category of all topological spaces and continuous maps. All subcategories of **Top** are assumed to be full and isomorphism-closed. Let **A** be an epireflective subcategory of **Top**. It is interesting to study closed hereditary coreflective subcategories in **A**. Recall that a subcategory is epireflective in **Top** if and only if it is closed under the formation of subspaces and topological products. A subcategory is coreflective in **A** if and only if it is closed under the formation of topological sums and extremal quotient objects. A coreflective subcategory of **A** will be called non-trivial if it contains a non-discrete space. We are interested in the following:

Question. Which non-trivial closed hereditary coreflective subcategories of **A** are minimal with this property?

In the talk we will answer the above question for certain epireflective subcategories **A** of **Top**.

The topological end space problem

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End spaces of infinite graphs sit at the interface between topology, graph theory, and group theory. They arise as the boundary of an infinite graph in a standard sense generalising the theory of the Freudenthal boundary developed by Freudenthal and Hopf in the 1940's for infinite groups.

A long-standing quest in infinite graph theory with a rich body of literature seeks to characterise the possible end structures of graphs, with the eventual goal of finding a purely topological characterisation of end spaces:

Question (The topological end space problem)**.** Find a topological characterisation of those spaces that arise as end space of some infinite graph.

In this talk, I will first explain our recent representation theorem for end spaces (see ArXiv:2111.12670 for details):

Theorem (Kurkofka & Pitz)**.** *Every end space is homeomorphic to the end space of a (certain canonical graph on a) special order tree.*

I will then describe possible directions of how our representation theorem may be used in attacking the topological end space problem.

Ideal boundedness of series vs Banach spaces possessing a copy of c_0

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Suppose that X is a Banach space. We will show that X does not contain an isomorphic copy of c_0 if and only if, for each series $\sum x_n$ which is not unconditionally convergent in X, the respective sets coding all bounded subseries and rearrangements are meagre. We use the Bessaga–Pełczyński c_0-embedding Theorem as a tool. Moreover, we prove a similar result for the ideal boundedness (assuming additionally $\liminf \|x_n\| = 0$ or $\limsup \|x_n\| = \infty$) and for the class of Baire ideals using Talagrand's characterisation.

[1] M. Balcerzak, M. Popławski, A. Wachowicz, *The Baire category of ideal convergent subseries and rearrangements*, Topology Appl. 231 (2017), 219–230.

[2] M. Popławski, *Ideal boundedness of subseries and rearrangements vs Banach spaces possessing a copy of* c_0, arXiv:1906.02449v1 (2019)

Big Ramsey degrees in the metric context

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Big Ramsey degrees are combinatorial invariants quantifiying the amount of information that needs to be added to a mathematical structure in order to get an analogue of the infinite Ramsey theorem in it. They are an active research topic in combinatorics, and until now, they have only been studied for discrete structures.

I will present a joint work in progress with A. Bartoš, T. Bice, J. Hubička, and M. Konečný in which we define a notion of big Ramsey degrees for metric structures. In this context, big Ramsey degrees, if they exist, are compact metric spaces. We are, for instance, able to prove that the Urysohn sphere has compact big Ramsey degrees.

If time permits, the case of uniform structures (without a metric) will also be discussed.

Continuity in right semitopological groups and semineighbourhoods of the diagonal

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Let X be a space and $P \subset X \times X$. We call P a *semineighbourhood of the diagonal* if $x \in \operatorname{Int} P(x)$ for all $x \in X$, where $P(x) = \{y \in G : (x, y) \in P\}$. A space X is called Δ-*nonmeager* (Δ_h-*;* Δ_s-*nonmeager*) if for any semineighbourhood of the diagonal P there exists a non-empty open $W \subset X$ such that that the condition (Δ) $W^2 \subset \overline{P}$ (respectively, (Δ_h) the set $\{x \in W : (x, y) \in P\}$ is dense in W for all $y \in W$; (Δ_h) the set $\{x \in W : (x, y) \in P$ for all $y \in W\}$ is dense in W) holds. A mapping $f : X \to Y$ is called *feebly continuous* if $\operatorname{Int} f^{-1}(U) \neq \emptyset$ for any open $U \subset Y$ such that $U \cap f(X) \neq \emptyset$.

Theorem. *[1] Let G be a regular right topological group such that every left shift $x \mapsto gx$ is feebly continuous. Let us assume that one of the following conditions is met: G is Δ-nonmeager and multiplication in G is feebly continuous; G is Δ_h-nonmeager and the operation $x \mapsto x^{-1}$ is continuous; G is Δ_s-nonmeager. Then G is a topological group.*

[1] E. Reznichenko, *Continuity in right semitopological groups*, 2022. Preprint, arXiv:2205.06316.

An Asplund space with norming M-basis that is not WCG

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We construct [1] an Asplund Banach space $\mathcal{X}$ with a norming Markuševič basis such that $\mathcal{X}$ is not weakly compactly generated. This solves a long-standing open problem from the early nineties, originally due to Gilles Godefroy. En route to the proof, we construct a peculiar example of scattered compact space, that also solves a question in Descriptive Topology due to Wiesław Kubiś and Arkady Leiderman.

[1] P. Hájek, T. Russo, J. Somaglia, and S. Todorčević, *An Asplund space with norming Markuševič basis that is not weakly compactly generated*, Adv. Math., 392 (2021), paper no. 108041, pp. 22.

The properties of tightness type of space of the permutation degree

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Let k be an infinite cardinal. We say that a subset F of X is k-closed (in X) if for every $B \subset F$ with $|B| \leq k$, the closure in X of the set B is contained in F [1].

Let k be an infinite cardinal, X and Y topological spaces. A map $\varphi\colon X \to Y$ is said to be k-continuous if for every subspace A of X such that $|A| \leq k$, the restriction $\varphi|A$ is continuous [1].

The tightness $t(x, X)$ of x in X is defined by $t(x, X) = \min\{k :$ for any set $A \subset X$ with $x \in \overline{A}$, there exists a subset B of A such that $|B| \leq k$ and $x \in \overline{B}\}$, and define the tightness $t(X)$ of X by $t(X) = \sup\{t(x, X) : x \in X\}$ [1].

The functional tightness of a space X is $t_0(X) = \min\{k : k$ is an infinite cardinal and every k-continuous real-valued function on X is continuous$\}$ [1].

The space of the permutation degree is given in [2].

Theorem. *If a set F is k-closed in a compact space X, then the set $SP^n_G F$ is k-closed in $SP^n_G X$.*

Theorem. *Let X be a compact space, $\bar{x} = (x_1, x_2, \ldots, x_n) \in X^n$ and $[\bar{x}] = \pi^s_{n,G}(\bar{x})$. If $t(SP^n_G X) \leq k$ and $t_0(\bar{x}, (\pi^s_{n,G})^{-1}[\bar{x}]) \leq k$, then $t_0(\bar{x}, X^n) \leq k$ and $t_0(x_i, X) \leq k$ for every $i = 1, \ldots, n$.*

[1] A. V. Arkhangelskii, *Topological Function Spaces*, Mathematics and Its Applications, Vol. 78 (1992), pp. 205.

[2] V. V. Fedorchuk and V. V. Filippov, *Topology of hyperspaces and its applications*, Mathematica, cybernetica. Moscow, 1989, pp. 48.

Weak* derived sets

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The weak* derived set $A^{(1)}$ of a subset A of a dual Banach space X^* is the set of weak* limits of bounded nets in A. It is well known that a convex subset of a dual Banach space is weak* closed if and only if it equals its weak* derived set. In genaral, taking weak* derived set is not an idempotent operation – it can happen that $A^{(1)}$ is a proper subset of $\left(A^{(1)}\right)^{(1)}$. This inspires the definition of iterated weak* derived sets. The order of A is then defined to be the least ordinal for which the iteration stabilizes. M. Ostrovskii provided the complete description of possible orders of subspaces of duals of separable non-quasi-reflexive spaces. In this talk we will present some partial results concerning orders of convex subsets of duals of non-reflexive spaces. We also present another special result motivated by the study of extension problems for holomorphic functions on dual Banach spaces. We show that for any non-quasi-reflexive Banach space X containing an infinite-dimensional subspace with separable dual and for any countable non-limit ordinal α we can always find a subspace A of X^* such that $A^{(\alpha)}$ is a proper norm dense subspace of X^*.

Čech and Katětov covering dimensions and more or less related questions concerning F-groups

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There are two covering dimensions, dim in the sense of Čech ($\dim X \le n$ if any finite open cover of X has a finite open refinement of order $\le n$) and $\dim_0$ in the sense of Katětov ($\dim_0 X \le n$ if any finite cozero cover of X has a finite cozero refinement of order $\le n$). It is proved that the covering dimension of the Sorgenfrey plane $S \times S$ is infinite, while, as is well known, $\dim_0 S^\kappa = 0$ for any cardinal κ. Examples of topological groups with similar properties are constructed, including a separable precompact Boolean group G with linear topology (generated by open subgroups) with $\dim_0 G^\kappa = 0$ and $\dim G = \infty$.

The open problem of the existence in ZFC of topological groups whose underlying space is an F-space (i.e., a space in which any two disjoint cozero sets are functionally separated) not being P-spaces is touched on. It is proved that the existence of an Abelian F-group G such that $\dim_0 G < \infty$ and $\psi(G) \le \omega$ is equivalent to the existence of a Boolean group with the same properties and that the existence of an Abelian F'-group (or of an extremally disconnected group) G with linear topology which is not a P-space implies the existence of a group of cardinality $\le 2^\omega$ with the same properties.

Question. Is it true that $\dim_0 X \le \dim X$ for any completely regular space X?

Cook continua as a tool in topological dynamics

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A Cook continuum C is a nondegenerate metric continuum such that for every subcontinuum K and every continuous map $f : K \to C$ either f is constant or $f(x) = x$ for all $x \in K$.

One can hardly imagine that Cook continua could be useful in topological dynamics, say for constructing spaces admitting an interesting nontrivial dynamics. We will controvert this by indicating that they can be glued together to obtain continua giving an answer to a problem in the theory of topological sequence entropy (for details see [1]).

[1] L. Snoha, X. Ye, and R. Zhang, *Topology and topological sequence entropy*, Sci. China Math., 63 (2020), pp. 205–296.

Structure of groups of order-preserving homeomorphisms of product spaces with lexicographic order and their compactifications

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Let X and Y be linearly ordered spaces, $H_+(X)$ and $H_+(Y)$ are their groups of order-preserving homeomorphisms. Let $\mathrm{Hom}_+^\partial(X)$ be the group $H_+(X)$ in the permutation topology and $\mathrm{Hom}_+(Y)$ be the group $H_+(Y)$ in the topology of pointwise convergence.

Theorem. *If each order preserving homeomorphism of the product $X \times Y$ with lexicographic order maps every fiber $\{x\} \times Y$ onto fiber $\{x'\} \times Y$, then the group of order-preserving homeomorphisms of the product $X \times Y$ with lexicographic order in the topology of pointwise convergence is topologically isomorphic to the semi-direct topological product $G \ltimes T$ where G is the power $\mathrm{Hom}_+(Y)^X$ in Tikhonoff topology and T is $\mathrm{Hom}_+^\partial(X)$.*

Corollary. *The group of order-preserving homeomorphisms in the topology of pointwise convergence of*

(1) *the lexicographically ordered square* $\mathbf{K}$ *is topologically isomorphic to* $\mathrm{Hom}_+([0,1])^{[0,1]} \ltimes \mathrm{Hom}_+^\partial([0,1])$;

(2) *the lexicographically ordered square of the real line* $\mathbf{R}$ *is topologically isomorphic to* $\mathrm{Hom}_+(\mathbb{R})^{\mathbb{R}} \ltimes \mathrm{Hom}_+^\partial(\mathbb{R})$;

(3) *the "Double Arrow" Alexandroff space* $\mathbf{D}$ *is topologically isomorphic to* $\mathrm{Hom}_+^\partial([0,1])$.

Countable tightness and the Grothendieck property in $C_p(X)$; other recent results

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A famous theorem of Grothendieck widely used in Analysis asserts that if X is a compact space and A is a subspace of $C_p(X)$, then the closure of A is compact if and only if every infinite subset of A has a limit point in $C_p(X)$. Casazza and Iovino used Grothendieck's Theorem to prove the undefinability of pathological Banach spaces in compact logics. C. Hamel and I extended their work to a large class of logics with type spaces satisfying Grothendieck's Theorem. Recently we have extended these results to logics with countably tight type spaces. Arhangel'skiĭ extensively studied generalizations of Grothendieck's Theorem in a 1998 paper and raised several problems. We answer many of these, showing them undecidable in ZFC, for example whether Lindelöf countably tight spaces satisfy the conclusion of Grothendieck's Theorem. Our affirmative answer (true, for example, under PFA) is a dramatic improvement of the previous result that required, under MA_{ω_1}, in addition that all finite powers of X be Lindelöf. We also provide a counterexample under $V = L$ to the MA_{ω_1} conclusion. The proofs require too much C_p-theory to do in a short talk; instead we mention other unpublished recent work that may be of interest. M. Morley proved in 1972 that the number of countable non-isomorphic models of a first-order theory is either countable, of size continuum, or of size $\aleph_1$. With C. Hamel, C. Eagle, and S. Müller, we prove that the analogous claim for second-order theories is undecidable. For those who do not attend my Toposym talk, let me mention that with Ivan Ongay Valverde, I have investigated various topological generalizations of descriptive set theory, most notably the class of upper semi-continuous compact-valued images of σ-projective sets of reals. These generalize the K-analytic spaces and a number of results about those spaces (e.g. regarding Selection Principles) do generalize, assuming large cardinals which imply determinacy axioms.

[1]Supported by NSERC grant A-7354

Some pseudocompact-like properties in certain topological groups

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In 2014, García-Ferreira and Ortiz-Castillo introduced the concept of *selective pseudocompactness* for topological spaces. This notion is stronger than pseudocompactness for topological groups, as demonstrated by García-Ferreira and Tomita in 2015.

In this talk we will present the following results:

(1) there exists a selectively pseudocompact group which is not countably pracompact;

(2) assuming the existence of a single selective ultrafilter, there exists a group which has all powers selectively pseudocompact but is not countably pracompact;

(3) there exists a countably compact group without non-trivial convergent sequences of size $2^{\mathfrak{c}}$.

The result (3) answers a question of Bellini, Rodrigues and Tomita, and its proof is a slight modification of a proof done in [1].

[1] M. Hrušák, J. van Mill, U. A. Ramos-García, and S. Shelah, *Countably compact groups without non-trivial convergent sequences*, Trans. Amer. Math. Soc., 374 (2021), pp. 1277–1296.

[1] The first author was supported by *São Paulo Research Foundation* (FAPESP), grant 2019/12628-0

[2] The second author was supported by *São Paulo Research Foundation* (FAPESP), grant 2021/00177-4

On a new problem of complexity theory arising from Galois–Tukey connections

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In [2] we have introduced a formalism which extracted ideas in many proofs of inequalities between cardinal characteristics of continuum. Andreas Blass in [1] observed that the formalism is categorical and present also in complexity theory.

Here we build on this understanding of complexity problems. To make the Blass' category fully viable we need to correctly extend the range of responses by an element "nar = no acceptable response". That is, an algorithm solving e.g. $3SAT^{nar}$ search problem has to halt also on unsatisfiable formulas with correct answer "nar".

Question. Find a place of $3SAT^{nar}$ problem in the hierarchy of computational complexity problems.

[1] A. Blass, *Questions and Answers. A Category Arising in Linear Logic, Complexity Theory, and Set Theory*, Advances in Linear Logic, Girard, JY. et al eds., (1995), pp. 61–81.

[2] P. Vojtas, *Generalized galois-tukey-connections between explicit relations on classical objects of real analysis*, Israel Math. Conf. Proc., 6 (1993), pp. 619–643.

[1] The author was supported by Dpt. Software Engineering, Charles University

On sequentiality of Polish topologies

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Gutierres showed in ZF that if the usual topology on the real line is sequential, then every infinite set of reals has a countable infinite subset. Using the method of balanced forcing, I show that the opposite implication is not provable, answering his question.

Theorem. *Relative to an inaccessible cardinal, it is consistent with ZF that every infinite set has a countable infinite subset, yet the topology of the real line is not sequential.*

The ease of the proof suggests that one may be able to (consistently with ZF) separate sequentiality of topologies on various Polish spaces. For example:

Question. Is it consistent with ZF that the topology of Euclidean line is sequential, while that of the Euclidean plane is not?

[1]The author was partially supported by grant EXPRO 20-31529X.

An embedded circle into $\mathbb{R}^3$ might not be able to escape before an isotoped linked circle

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The mathematically precise statement of the problem that was intuitively described in the title is following isotopy-extension problem: Given two linked embedded circles and an isotopy for one of them, is it possible to extend the embedding of the second circle to an isotopy, so that at any time of the isotopy both circles remain disjoint, in particular in the case, where the second circle is just parallel to a meridinal curve to the first one? – Since, when isotopying a circle, it cannot bump into itself or just shrink via very small circles down to a point, the result as described in the title is a bit counter-intuitive. However in the first half of the talk a corresponding example will be constructed, based on a construction trick that has already been used to construct Alexander's horned sphere. In the second half of this talk I want to introduce the problem that made me ask this isotopy-extension question: It is the problem of deciding, whether there exist knots (and it is clear that at most totally wild knots might have such a property) that even with respect to ordinary (not necessarily ambient) isotopy are non-equivalent to the trivial knot. In particular the consequences that the newly discovered example has for attacking this problem shall be discussed.

The large-scale geometry of LCA groups

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Large-scale geometry, also known as coarse geometry, is the branch of mathematics that studies global, large-scale properties of spaces. Since the breakthrough work of Gromov, large-scale geometry has played a prominent role in geometric group theory and, in particular, in the study of finitely generated groups and their word metrics. This large-scale metric approach was successfully extended up to the class of locally compact σ-compact groups by Cornulier and de la Harpe. To study more general topological groups, coarse structures, introduced by Roe as the large-scale counterpart of uniformities, are required. In this presentation, we focus on the compact-group coarse structure, induced by the family of compact subsets, and the left-coarse structure, introduced by Rosendal, in the class of locally compact abelian groups.

During the first part of the talk, we present results concerning the compact-group coarse structure, emphasising the role of Pontryagin duality as a bridge between topological properties and their large-scale counterparts. Then, we discuss the relation between the compact-group coarse structure and the left-coarse structure, showing that they coincide in this class. To conclude, we apply this result to the theory of Banach spaces.

On convex structures in quasi metric spaces

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Künzi and Yilzid introduced the concept of convexity structures in the sense of Takahashi in quasi-pseudometric spaces. In this talk, we continue the study of this theory, introducing the concept of W-convexity for real-valued pair of functions defined on an asymmetrically normed real vector space. Moreover, we show that all minimal pairs of functions defined on an asymmetrically normed real vector space equipped with a convex structure which is W-convex whenever W is translation-invariant.

POSTERS

Three posters are presented at Toposym 2022. This book contains only abstracts of posters submitted before June 27, 2022. All posters are accompanied by oral presentations.

Convex realizations of neural codes in one dimension

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Neural codes are collective activity of the neurons which are electrically active cells in our brain. In this paper, we study the openness and closeness of the convex neural codes in one dimension. We discuss the possibilities of a convex code to be realized in dimension 1. That is, conditions for the minimal embedding dimension of a neural code to be 1.

Projections of almost connected groups as G-fibrations

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A G-fibration is the equivariant version of a Hurewicz fibration, that is, an equivariant map with the right lifting property with respect to the G-embeddings $X \times \{0\} \hookrightarrow X \times I$.

A well known result about G-fibrations states that if H is a closed subgroup of a compact Lie group G, then any G-map $p : E \to G/H$ is a G-fibration. A natural question is whether this result remains valid when working with a non-compact or non-Lie acting group. To answer this, we are going to give generalizations of some classical results that lead us to prove that p is also a G-fibration whenever G is a (not necessarily compact) Lie group or an almost connected metrizable group and H its compact subgroup.

On dense sets of products of spaces

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The classical Hewitt–Marczewski–Pondiczery theorem states that if $d(X_s) \leqslant \tau$ $(\omega \leqslant \tau)$ for every $s \in S$ and $|S| \leqslant 2^\tau$ then $d(\prod_{s \in S} X_s) \leqslant \tau$.

Very important is the problem of the existence of a dense set of a cardinality τ in the product $\prod_{s \in S} X_s$ which contains no convergent nontrivial sequences.

For $\tau = \omega$ the existence of such set were proved for $I^{\mathfrak{c}}$ (W.H. Priestly, 1970), for $D^{\mathfrak{c}}$, where D is the two point discrete space (P. Simon, 1978), for $Z^{\mathfrak{c}}$, where Z is separable not single point T_1-space (A. Gryzlov, 2018), for a product of $2^{\mathfrak{c}}$ separable decomposable spaces, i.e. spaces, which contain two not empty closed disjoint sets (A. Gryzlov, 2020).

We prove

Theorem. *For a regular cardinal τ the product $\prod_{\alpha \in 2^\tau} X_\alpha$ of decomposable spaces, where $d(X_\alpha) = \tau$ $(\alpha \in 2^\tau)$, contains a dense set Q, $|Q| = \tau$, such that in every subset $P \subseteq Q$, $|P| = \tau$, there is $P' \subseteq P$, $|P'| = \tau$, which contains no convergent nontrivial sequences and therefore is sequentially closed.*

[1] This work carried within the framework of state assigment of Ministry of Science and Higher Education of Russia (FEWS-2020-0009).

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