ZFC solution to 9 problems of Tkachuk on functional countability

Peter Nyikos

nyikos@math.sc.edu

A Tychonoff space is called *functionally countable* if every real-valued continuous function has countable range. A compact space is functionally compact iff it is scattered. At this year's STDC conference, Vladimir Tkachuk posed 14 problems on functional countability. A ZFC solution is given for 9 of these problems, of which the most demanding is Question 8: If X is a compact, Fréchet–Urysohn space such that $X^2 \setminus \Delta_X$ is functionally countable, must X be separable? The counterexample has 2^ω isolated points and is the one-point compactification of a finer topology on $(0,1)$ such that the non-isolated points form a closed discrete subspace.