Recognizing the topologies on subspaces of L^p-spaces on metric measure spaces

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As applications of the theory of infinite-dimensional manifolds, topological structures of function spaces have been researched, and “typical” convex sets in Hilbert space ℓ_2 have been recognized among them. For instance, the linear span ℓ_2^f of the orthonormal basis of ℓ_2 is detected in several function spaces as a factor. In this talk, we shall study the topological types with ℓ_2^f as a factor of subspaces of L^p-spaces on metric measure spaces.

Suppose that X is a Borel-regular Borel metric measure space such that every open ball has a positive and finite measure. For $1 \leq p < \infty$, the L^p-space $L^p(X)$ on X is homeomorphic to ℓ_2 when X is infinite and separable. We investigate the topology on the subspace consisting of uniformly continuous maps in $L^p(X)$.

Characterizing compact sets in function spaces plays important roles in the study on them. By using average functions, we shall give a generalization of the Kolmogorov–Riesz theorem, which is an L^p-version of the Ascoli–Arzelà one. Applying that criterion, we recognize the topology on the subspace consisting of lipschitz maps with bounded supports in $L^p(X)$.