The double density spectrum of a topological space

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The set of densities of *all* dense subspaces of a topological space X is called the *double density spectrum* of X and is denoted by $dd(X)$.

We improve an earlier result by showing that $dd(X)$ is always ω-closed (i.e. countably closed) if X is Hausdorff. We characterize the double density spectra of Hausdorff and of regular spaces:

Let S be a non-empty set of infinite cardinals. Then

(1) $S = dd(X)$ for a Hausdorff space X if and only if S is ω-closed and $\sup S \leq 2^{2^{\min S}}$;

(2) $S = dd(X)$ for a regular space X if and only if S is ω-closed and $\sup S \leq 2^{\min S}$.

We do not have a characterization of the double density spectra of compact spaces but give some non-trivial consistency results concerning them:

(1) If $\kappa = cf(\kappa)$ embeds in $\mathcal{P}(\omega)/\text{fin}$ and S is a set of uncountable regular cardinals $< \kappa$ with $|S| < \min S$, then there is a compactum C such that $\{\omega, \kappa\} \cup S \subset dd(C)$, moreover $\lambda \notin dd(C)$ whenever $|S| + \omega < cf(\lambda) < \kappa$ and $cf(\lambda) \notin S$.

(2) It is consistent to have a separable compactum C such that $dd(C)$ is not ω_1-closed.

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