

## More on the properties of almost connected pro-Lie groups

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Projective limits of finite-dimensional Lie groups, so called *pro-Lie* groups, form a wide class of topological groups with nice properties. This class, defined by K.H. Hofmann and S. Morris some fifteen years ago, contains all Lie groups, is closed under arbitrary products and closed subgroups. All compact groups, locally compact Abelian groups, and connected locally compact groups are pro-Lie groups.

Following Hofmann and Morris, we say that a topological group  $G$  is *almost connected* if the quotient group of  $G$  by the connected component of its identity is compact. Almost connected pro-Lie groups constitute a proper subclass of pro-Lie groups. For example, it is easy to verify that every almost connected pro-Lie group is  $\omega$ -*narrow*, which means that the group can be covered by countably many translates of every neighborhood of its identity.

It turns out that all almost connected pro-Lie groups as well as their continuous homomorphic images are  $\mathbb{R}$ -*factorizable* and  $\omega$ -*cellular* (i.e. every family of  $G_\delta$ -sets contains a countable subfamily whose union is dense in the union of the whole family). We will also present a general result which implies, in particular, that if a topological group  $G$  contains a compact invariant subgroup  $K$  such that the quotient group  $G/K$  is an almost connected pro-Lie group, then  $G$  is  $\mathbb{R}$ -*factorizable* and  $\omega$ -*cellular*.

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