$\mathfrak{G}$-Bases in free objects of Topological Algebra

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A topological space X has a *local* $\mathfrak{G}$*-base* if every point $x \in X$ has a neighborhood base $(U_\alpha)_{\alpha \in \omega^\omega}$ such that $U_\beta \subset U_\alpha$ for all $\alpha \le \beta$ in ω^ω.

Theorem *For a Tychonoff space X the following conditions are equivalent:*

1. *The free Abelian topological group $A(X)$ of X has a local $\mathfrak{G}$-base.*
2. *The free Boolean topological group $B(X)$ of X has a local $\mathfrak{G}$-base.*
3. *The universal uniformity $\mathcal{U}(X)$ of X has a $\mathfrak{G}$-base.*

If X is first-countable and perfectly normal, then (1)–(3) are equivalent to:

4. *X is metrizable and has σ-compact set X' of non-isolated points.*

Theorem *For a Tychonoff space X the following conditions are equivalent:*

1. *The free locally convex space $L(X)$ of X has a local $\mathfrak{G}$-base.*
2. *The free topological vector space $V(X)$ of X has a local $\mathfrak{G}$-base.*
3. *The universal uniformity $\mathcal{U}(X)$ of X has a $\mathfrak{G}$-base and the function space $C(X)$ is ω^ω-dominated (in $\mathbb{R}^X$).*

The conditions (1)-(3) imply (and if X is not a P-space, are equivalent to):
(4) the free topological group $F(X)$ of X has a local $\mathfrak{G}$-base.
If X is first-countable and perfectly normal, then (1)–(3) are equivalent to metrizability and σ-compactness of X.

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