Toposym 2011
Book of Abstracts

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Preface

The first Topological Symposium took place in 1961 at a time when the world was still painfully divided during the Cold War. Communication across the border was difficult, although the situation had improved somewhat over the previous decade. In this state of things Eduard Čech decided to organize an event which would bring together mathematicians from the East and the West. It was an enormous effort on his part and, alas, he was not able to see the fruits of it — he died in 1960. However his efforts were not in vain. His students and colleagues managed to finish what he started and in 1961 147 mathematicians gathered in Prague for a week devoted to topology. This has started a tradition that every five years mathematicians from all over the world, interested in diverse areas of topology, come to meet in Prague.

The Toposym's have been numbered by natural numbers. And since most people think zero is an unnatural number, this year's meeting is the 11th, although this year we are celebrating 5*10 years of Toposym's. Here's to another 50 at least as successful years!

Jonathan Verner

INVITED LECTURES

From Tsirelson Space to the solution of the "scalar plus compact" problem.

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Tsirelson space appeared as the first ``truly non-classical'' Banach space. It is the first example of a reflexive space, not containing any $\ell_p$, $1 < p < \infty$. Thirty five years later (2007) the ``scalar plus compact'' problem was solved by R. Haydon and the speaker. More precisely, it is shown that there exists a Hereditarily Indecomposable Banach space $\mathfrak{X}$ with it's dual $\mathfrak{X}^*$ isomorphic to ℓ_1 and every bounded linear operator form $\mathfrak{X}$ to $\mathfrak{X}$ is of the from $\lambda I + K$ with λ a scalar, I the identity operator and K a compact operator. We shall overview the course towards the solution of the problem, which among others is based on the seminal work of J. Bourgain, W.T. Gowers and B. Maurey. We will also discuss some consequences and the fundamental structure of the norm of the space $\mathfrak{X}$.

Bishop-Phelps-Bollobás theorem and Asplund operators

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Topology is oftentimes a powerful and unavoidable tool to face some problems that naturally arise in functional analysis. The starting point for this lecture is the following definition:

Definition *(Acosta, Aron, García and Maestre, 2008) A pair of Banach spaces (X,Y) is said to have the Bishop-Phelps-Bollobás property (BPBp) if for any $\epsilon > 0$ there are $\eta(\epsilon) > 0$ and $\beta(\epsilon) > 0$ with $\lim_{t\to 0} \beta(t) = 0$, such that for any $T \in S_{L(X,Y)}$, if $x_0 \in S_X$ is such that $|T(x_0)| > 1 - \eta(\epsilon)$, then there are $u_0 \in S_X$ and $S \in S_{L(X,Y)}$ satisfying*

$$|S(u_0)| = 1, |x_0 - u_0| < \beta(\epsilon) \text{ and } |T - S| < \epsilon.$$

The notion above is a strengthening of the Bishop-Phelps property for operators introduced by J. Lindenstrauss in the 60's and studied thoroughly during the past decades. A Bollobás result states that for any Banach space X the pair $(X,\mathbb{R})$ has BPBp, in other words, BBBp always works for linear forms. A number of authors have studied BPBp for operators and most of the times known Bishop-Phelps properties for operators can be strenghten to BPBp. Nonetheless, it was unknown if for the classical space c_0 there were pairs (c_0,Y) with BPBp with Y infinite dimensional. We prove that this is the case for any $Y = C(K)$ with K compact. Our results are even better than this because by using the so-called Asplund operators we add a variety of more cases for BPBp that even allow us to replace $C(K)$ by the disk algebra $A(\mathbb{D})$. The success of our methods is based upon the notion of fragmentability (a very topological tool) that once again is useful in functional analysis.

This work is in collaboration with R. Aron and O. O. Kozhushkina.

[1] Research supported by FEDER and MEC grant MTM2008-05396 and by Fundación Séneca (CARM), grant 08848/PI/08.

Mackey groups and Mackey topologies

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Inspired by Mackey-Arens' theorem on locally convex topological vector spaces and its extension to locally precomapct groups [], Chasco, Martín-Peinador and Tarieladze [] introduced in 1999 the notion of *Mackey topology* τ_M of a locally quasi convex topological abelian group (G,τ) (namely, τ_M is the finest locally quasi convex topology on G having the same continuous characters as (G,τ)). The group (G,τ) is called *Mackey group*, if $\tau = \tau_M$. They showed that some classes of topological abelian groups (including among others the complete metrizable groups) are Mackey. Further progress in this topic was obtained in [],[], yet a complete description of the Mackey groups is not yet available even in the metrizable case. The talk will discuss these issues and the following question raised in []:

Question *Does every locally quasi convex topological abelian group* (G,τ) *admit a Mackey topology ?*

Efimov's problem revisited

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A topological space is an *Efimov space* if it is infinite, compact and contains no infinite converging sequence and no homeomorphic copy of $\beta\omega$. A Boolean algebra will be called Efimov if its Stone space of ultrafilters is an Efimov space. Efimov spaces are known to exist in various models of set-theory, for example, the continuum hypothesis implies they exist. We will give a brief review of these results. It is not known if they exist in ZFC, and, as was asked in the recent open problems book, it was not even known if their existence was consistent with Martin's Axiom and the failure of CH. We prove that the hypothesis $\mathfrak{b} = \mathfrak{c}$ is sufficient to imply their existence. We use Koppelberg's concept of minimally generated Boolean algebras and uncover a surprising connection with the Scarborough-Stone problem.

Question *Do Efimov spaces exist in ZFC?*

Question *Is there a minimally generated Boolean algebra which is Efimov?*

[2] partially supported by NSF

On Hilbert Dynamical Systems

Eli Glasner

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I will present a joint work with Benjy Weiss where we analyze Hilbert dynamical systems. In particular two corollaries of this analysis will ensue: Returning to a classical question in Harmonic Analysis we strengthen an old result of Walter Rudin. We show that there exists a weakly almost periodic function on the group of integers $\mathbb{Z}$ which is not in the norm-closure of the algebra $B(\mathbb{Z})$ of Fourier-Stieltjes transforms of measures on the dual group $\hat{\mathbb{Z}} = T$, and which is *recurrent*. We also show that there is a Polish monothetic group which is reflexively but not Hilbert representable.

Markushevich bases in Banach spaces

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We survey some classical and new results and applications concerning biorthogonal systems, in particular Markushevich bases in Banach spaces. We sketch a proof of the following recent result due to the authors (generalizing the work of Plichko).

Theorem *If a Banach space X has a Markushevich basis then X has a* 5*-bounded Markushevich basis.*

A non-chainable plane continuum with span zero

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In 1964, A. Lelek gave a proof that any two continuous maps f,g from any continuum (compact connected metric space) C to a chainable continuum X with $f(C) = g(C)$ must have a coincidence point (i.e. a point p in C with $f(p) = g(p)$). Lelek later asked whether this property, called *span zero*, characterizes chainability of the continuum X.

I will outline the construction of a counterexample for this question, and discuss implications regarding the classification of homogeneous plane compacta.

Variations on ω-boundedness

I. Juhász

Let $\mathcal{P}$ be a property (or, equivalently, a class) of topological spaces. A space X is called *$\mathcal{P}$-bounded* if every subspace of X with (or in) $\mathcal{P}$ has compact closure. Thus, countable-bounded has been known as ω-bounded and (σ-compact)-bounded as strongly ω-bounded.

Our aim is to study the interrelations of these two well-known "boundedness" concepts with $\mathcal{P}$-boundedness where $\mathcal{P}$ is one of the further countability properties *weakly Lindelöf, Lindelöf, hereditarily Lindelöf,* and *ccc*. Here is a list of some of our main results in which all spaces are Tychonov:

1. There is an ω-bounded space which is not HL-bounded.
2. If there is a compact L space then there exists a normal, locally compact, and first countable space which is ω-bounded but not HL-bounded.
3. There is a locally compact space that is HL-bounded but not ccc-bounded.
4. There is a 0-dimensional, normal, first countable, and locally compact space that is ccc-bounded but not (σ-compact)-bounded.
5. There is a (σ-compact)-bounded space that is neither ccc-bounded nor L-bounded, and under CH it is not even HL-bounded.

The following question remains open:

Question *Is there an L-bounded space that is not wL-bounded or even not ccc-bounded?*

This is joint work with Jan van Mill and Bill Weiss.

On typical properties of Hilbert space operators

Tamás Mátrai

What are the typical properties of Hilbert space operators, in the sense of Baire category? And anyway, why is this problem interesting?

To start with the second question, our answer includes such observations as:

1. there is no perfect analogue of the Lebesgue measure in infinite dimensional Banach spaces, so meagerness may be the only natural notion of smallness which can come to the help of the analyst;

2. while the theory of e.g. unitary, self-adjoint, or merely normal operators is rich and well-understood, general Hilbert space operators are considered "hard to study".

The answer to the first question, obviously, may depend on the underlying topology. So in collaboration with Tanja Eisner, we investigated the typical behavior of Hilbert space operators in the norm topology and in four important separable topologies (a property Φ of operators is *typical* if the operators satisfying Φ form a co-meager set).

We obtained that in the separable topologies, from the point of view of Baire category, the theory of Hilbert space operators reduces to the theories of very particular classes of operators, e.g. unitary operators, positive self-adjoint operators or even one single operator. In the norm topology, the theory of Hilbert space operators does not trivialize; however, this topology is so fine that every property we studied, e.g. various mapping and spectral properties, holds for a non-meager set of operators. In a sense, our results outline some limitations of Baire category methods in operator theory.

Uniquely Universal Sets

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We say that $X \times Y$ satisfies the Uniquely Universal property (UU) iff there exists an open set $U \subseteq X \times Y$ such that for every open set $W \subseteq Y$ there is a unique cross section of U with $U_x = W$. Michael Hruák raised the question of when does $X \times Y$ satisfy UU and noted that if Y is compact, then X must have an isolated point. We consider the problem when the parameter space X is either the Cantor space 2^ω or the Baire space ω^ω.

Theorem *If Y is a locally compact Polish space which is not compact, then $2^\omega \times Y$ has UU.*

Theorem *If Y is Polish, then $\omega^\omega \times Y$ has UU iff Y is not compact.*

Theorem *If Y is a σ-compact subset of a Polish space which is not compact, then $\omega^\omega \times Y$ has UU.*

Laminations and Complex Dynamics

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A lamination is closed set of chords (called leaves) in the unit disk so that no two intersect inside the open unit disk. Invariant laminations (under the covering map $\sigma_d(z) = z^d$ in the complex plane) were introduced by Thurston as a means to study the dynamics of individual polynomials acting on the complex plane and the space of all polynomials with connected Julia sets.

In this talk we will explore the connection between laminations and (invariant) equivalence relations on the unit circle. Although all such equivalence relations determine a lamination the reverse conclusion is more complicated. We will also discuss the connection between laminations and the dynamics of polynomials acting on the complex plane.

Selective separability of Pixley-Roy hyperspaces

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A space X is said to be *selectively separable* (=*M-separable*) if for every sequence $\{D_n : n \in \omega\}$ of dense subsets of X, there are finite sets $F_n \subset D_n$ $(n \in \omega)$ such that $\bigcup\{F_n : n \in \omega\}$ is dense in X. This notion was first introduced by M. Scheepers, and later studied systematically by A. Bella, M. Bonanzinga, M.V. Matveev and V.V. Tkachuk in 2008.

It is known that consistently there are two selectively separable spaces X and Y such that $X \times Y$ is not selectively separable. Such examples were constructed by G. Gruenhage, L. Babinkostova, D. Repovš and L. Zdomskyy, A. Dow and D. Barman respectively.

We show that the Pixley-Roy hyperspace $PR(X)$ of a space X is selectively separable if and only if X is countable and every finite power of X has countable fan-tightness for finite sets. As an application, under $\mathfrak{b} = \mathfrak{d}$ there are selectively separable Pixley-Roy hyperspaces $PR(X), PR(Y)$ such that $PR(X) \times PR(Y)$ is not selectively separable.

[3] The author was supported by KAKENHI (No. 22540154)

Metrizability of compact groups via conditions on their dense subgroups

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For an abelian topological group G, we denote by $\widehat{G}$ the dual group of all continuous characters of G endowed with the compact-open topology. Following Comfort, Raczkowski and Trigos-Arrieta, we say that a dense subgroup D of G *determines* G if the restriction homomorphism $\widehat{G} \to \widehat{D}$ of the dual groups is a topological isomorphism, and we say that G is *determined* if every dense subgroup of G determines G. Chasco and Außenhofer proved that every metrizable abelian group is determined. A remarkable partial inverse of this result is due to Hernández, Macario and Trigos-Arrieta: Every compact determined abelian group is metrizable. (Under CH, this was established earlier by Comfort, Raczkowski and Trigos-Arrieta.) Answering a question of Hernández, Macario and Trigos-Arrieta, we prove (in ZFC) that *a compact abelian group determined by all its dense pseudocompact subgroups is metrizable*. (Under CH, the same statement was proved recently by Bruguera, Chasco, Domínguez, Tkachenko and Trigos-Arrieta.) Under CH, we prove a stronger version of this theorem saying that every compact abelian group determined by all its dense countably compact subgroups is metrizable. A main technical tool in the proof of main theorem is a ZFC construction of "sufficiently many" pseudocompact abelian groups of weight ω_1 having all their compact subsets metrizable.

This is a joint work with Dikran Dikranjan.

L_0 and its representations

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Groups of the form $L_0(\mu,H)$ of all measurable functions from a measure space of μ to a compact group H with the topology of convergence in measure have recently been realized to be relevant to various questions in topological and measurable dynamics. For example, the groups $L_0(\mu,\mathbb{T})$, where $\mathbb{T}$ is the circle group, turned out to be of interest in the study of generic elements of large topological groups like the unitary group or the automorphism group of a measure. I will survey various interconnected results involving L_0 groups and present a new theorem classifying all unitary representations of $L_0(\mu,\mathbb{T})$ and some of its consequences.

Feebly compact paratopological groups

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There are several structures in Topological Algebra more general than that of a topological group. In recent years, many efforts has been done in the study of *paratopological groups*, i.e., groups with topology that makes multiplication jointly continuous. It is known that completely metrizable or locally compact paratopological groups are *topological groups.* More generally, according to Bouziad's theorem, every Čech-complete *semitopological group* (a group with separately continuous multiplication) is a topological group. In all of these results, the corresponding paratopological or semitopological groups are explicitly assumed to be Tychonoff. The same happens with Reznichenko's theorem: Every pseudocompact paratopological group is a topological group.

Spaces in which every infinite family of open sets has an accumulation point are called *feebly compact.* Clearly feeble compactness and pseudocompactness coincide in Tychonoff spaces. It is natural, therefore, to study feebly compact paratopological groups. In 2004 Arhangel'skii and Reznichenko proved that a feebly compact *regular* paratopological group is a topological group (see section 2.4 of []), thus generalizing Reznichenko's theorem. Hence, the first question to ask is whether feebly compact Hausdorff paratopological groups must be topological groups. Ravsky answered this question in the negative. Afterwards he used Martin's axiom to construct an example of a countably compact Hausdorff paratopological group with discontinuous inverse []. These and other results revealed a tight relation between the concepts of feeble compactness, 2-*pseudocompactness* (coming from the theory of bitopological spaces), and precompactness in paratopological groups.

In the lecture we are going to explain in detail the interplay between the above mentioned concepts and present several examples of feebly compact Hausdorff paratopological groups that enable us to answer several problems posed by Romaguera, Sanchis, and Ravsky.

Clones of topological spaces

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Clones are widely used in universal algebra, sometimes under the name algebraic theory (monosorted, finitary see F. W. Lawvere [],[]). It was a merit of John Isbell and Walter Taylor with his monograph "Clone of topological space" [] who shifted this important and interesting field of problems also in topology.

Let us present the definition:

Definition *A clone CloX of a topological space X, is a category of all its finite powers and all their continuous maps.*

John Isbell attracted the attention to this notion by the problem whether there exist two topological spaces with isomorphic monoids of all continuous selfmaps and non-isomorphic clones. This problem was then solved positively within the class of all metrizable spaces. Then the solution inspired many other questions, e.g. some connections between rigidities and semirigidities; a connection between continuous and uniformly continuous maps; the relations of elementary equivalence of clones and the isomorphisms. These results inspired further problems, e.g. the existence of non-homeomorphic spaces X and Y with isomorphic clones and such that $X^n = Y^n$ for all $n \in \omega$, $n>1$ or $Y = X^2 = X^6$.

In the lecture, all these facts will be presented more in detail.

Generic representations of abelian groups

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If Γ is a countable group and G a Polish group, denote by Hom(Γ,G) the Polish space of all homomorphisms $\Gamma \to G$. When G is the group of symmetries of a certain object X, Hom(Γ,G) can be regarded as the space of all actions of Γ on X that preserve the relevant structure. In this talk, we will concentrate on three examples for G: the unitary group of an infinite-dimensional Hilbert space, the automorphism group of a standard probability space, and the isometry group of the Urysohn metric space. We are interested in the general question: what are the generic properties in Hom(Γ,G), that is the properties that hold on a comeager set? This type of question has a long history in ergodic theory (mostly in the classical situation when $\Gamma = \mathrm{Z}$).

We concentrate on the case where the group Γ is abelian and investigate the generic properties of the closed subgroup $\overline{\pi(\Gamma)}$. Among our results are that under mild assumptions on Γ, the generic $\overline{\pi(\Gamma)}$ is extremely amenable as well as that the generic properties of $\overline{\pi(\Gamma)}$ do not depend on Γ as long as it is torsion-free. We also obtain a new proof of the extreme amenability of our three examples for G.

Combinatorial properties of open covers of products

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One of the central questions in topology is to detect which properties are preserved by finite products. In our talk we shall focus on combinatorial properties of open covers of topological spaces. The typical examples of these are the covering properties of Menger, Scheepers, Hurewicz, and Rohberger, the lindelöfness, being a γ-space, etc. In particular, we shall discuss productive versions of these properties (a topological space X is said to be productively $\mathcal{P}$, where $\mathcal{P}$ is a property, if the product $X \times Y$ satisfies the property $\mathcal{P}$ provided so does the space Y), consider examples constructed with the help of scales, and show how these properties are related to Michael spaces and D-spaces.

[4] The author was supported by the FWF grant M 1244-N13

Topological Homogeneity

Jan van Mill

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Topological homogeneity is not a well understood notion. As far as we know, there is only one known ZFC example of a homogeneous compact space which is not a product of dyadic spaces and first countable spaces. We discuss several classes of homogeneous spaces, among them the countable dense homogeneous spaces and the uniquely homogeneous spaces. We also state some intriguing open problems, some of which are (very) old and some of which came from recent investigations.

CONTRIBUTED LECTURES

Small Whitney Blocks

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Given a metric continuum X, let $C(X)$ be its hyperspace of subcontinua. Given a Whitney map $\mu : C(X) \to [0,1]$ and a number $t \in (0,1)$, $\mu^{-1}([0,t])$ is called a Whitney block, which is a subcontinuum of $C(X)$. In this talk we give some partial answers to the next questions:

Question *Let P a topological property. If a continuum X has property P, then their Whitney blocks have property P?*

Question *If the Whitney blocks have a topological property, is it true that the space X has the same property?*

Among others, we have considered the following properties: aposyndesis, contractibility, being absolute neighborhood, having trivial fundamental group, unicoherence, the property of Kelley and the fix point property.

Trotter-Kato type results for second order differential inclusions

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Some Trotter-Kato type results will be presented for a class of second order difference inclusions in a real Hilbert space. The inclusion contains a nonhomogeneous term f and is governed by a nonlinear operator A, which is supposed to be maximal monotone and strongly monotone. The associated boundary conditions are also of monotone type. One shows that, if A^n is a sequence of operators which converges to A in the sense of resolvent and f^n converges to f in a weighted l^2- space, then under additional hypotheses, the sequence of the solutions of the difference inclusion associated to A^n and f^n is uniformly convergent to the solution of the inclusion associated to A and f.

[5] The second author was partially supported by CNCSIS

The largest classes of closed sets for coincidence of hypertopologies

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Let (X,d) be a metric space and $\mathcal{C}\mathrm{l}(\mathcal{X})$ the family of closed subsets of X. We search the "largest" family $\mathcal{A} \subset \mathcal{C}\mathrm{l}(\mathcal{X})$ such that different pairs of well known hypertopologies coincide on $\mathcal{A}$. The hypertopologies which we study here are Hausdorff, Attouch-Wets, Vietoris, bounded Vietoris, proximal and locally finite topologies.

The p–adic topologies on $\mathbb{Z}$ are not Mackey topologies

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In this talk we sketch a proof for the fact that to every non-discrete Hausdorff linear topology on $\mathbb{Z}$ there exists another metrizable locally quasi-convex group topology which is strictly finer than the linear topology and such that the character groups coincide. Applying this result to the p-adic topologies on $\mathbb{Z}$, we give a negative answer to a question of Dikranjan, whether these topologies are Mackey.

Measurability in $C(2^\kappa)$ and Kunen cardinals

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We prove that the Baire and the Borel σ-algebra coincide on the Banach space $C(2^\Gamma)$ if and only if $|\Gamma|$ is a Kunen cardinal, which means that all subsets of $\Gamma \times \Gamma$ belong to the σ-algebra generated by product sets. The cardinality of the continuum is a Kunen cardinal under Martin's Axiom. Our result generalizes a result by Fremlin about the space $\ell_1(\Gamma)$.

[6] The first author was partially supported by ESF
[7] The first author was partially supported by the NSF
[8] The first author was partially supported by ESF

The Shape and Cohomology Exact Sequences of a Map

Vladimer Baladze

In talk the shape of continuous map $f : X \to Y$ is defined (cf.[]). Applying Čech and Vietoris constructions then it will be shown that exist two equivalence functors from the category of maps of topological spaces to the pro-category of category of maps of CW-complexes and to the pro-category of appropriate homotopy category of maps of CW-complexes. Next we will give the definitions of functors from the category of maps to the category of long exact sequences of normal homology pro-groups and to the category of long exact sequences of normal cohomology inj-groups. Using the result of ([], [], []) we will prove the following theorems.

Theorem *For each map $f : X \to Y$ of topological spaces and abelian group G there exist the long exact sequences of normal homology pro-groups and normal cohomology inj-groups*

$$\ldots \to pro - H_n(X;G) \to pro - H_n(Y;G) \to pro - H_n(f;G) \to \ldots$$

$$\ldots \to inj - H^n(f;G) \to inj - H^n(Y;G) \to inj - H^n(X;G) \to \ldots,$$

where

$pro{-}H_n(f;G) = \{H_n(f_{\alpha\beta\nu};G)\}_{(\alpha,\beta,\nu)\in \mathrm{cov}_N(f)}$, $inj{-}H^n(f;G) = \{H^n(f_{\alpha\beta\nu};G)\}_{(\alpha,\beta,\nu)\in \mathrm{cov}_N(f)}$, $H_n(f_{\alpha\beta\nu};G) = H_n(Cyl(f_{\alpha\beta\nu})), X_\beta;G)$, $H^n(f_{\alpha\beta\nu};G) = H^n(Cyl(f_{\alpha\beta\nu})), X_\beta;G)$, $f_{\alpha\beta\nu} : X_\beta \to Y_\alpha$, $\nu : \beta > f^{-1}(\alpha)$, $\alpha \in \mathrm{cov}_N(Y)$, $\beta \in \mathrm{cov}_N(X)$ *and* $\mathrm{cov}_N(X)$ *and* $\mathrm{cov}_N(Y)$ *are the sets of normal open coverings of X and Y, respectively.*

Theorem *For each map $f : X \to Y$ of topological spaces there exists a long exact sequences of normal cohomology groups*

$$\ldots \to \check{H}^n(f;G) \to \check{H}^n(Y;G) \to \check{H}^n(X;G) \to \ldots,$$

where $\check{H}^n(f;G) = \lim_{\to} inj - H^n(f;G)$.

Corollary *([]). For each pair* (X,A) *of topological spaces there exists a long exact sequence of normal cohomology groups*

$$\ldots \to \check{H}^n(i;G) \to \check{H}^n(Y;G) \to \check{H}^n(X;G) \to \ldots,$$

where i *is the inclusion map* $i : A \to X$.

A note on coclones of topological spaces

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The clone of a topological space is known to have a strictly more expressive first-order language than that of the monoid of continuous self-maps. The talk will discuss expressiveness of the first-order language of coclones of topological spaces instead (i. e. clones in the category dual to that of topological spaces and continuous maps). We will show that, in contrast to clones, the first-order properties of coclones cannot express anything more than those of the monoid, except for the case of discrete and indiscrete spaces. Specifically, the following results will be presented:

Theorem *Suppose X and Y are non-indiscrete spaces. Further, suppose that $Mon(X)$ is isomorphic to $Mon(Y)$. Then $Coclo(X)$ is isomorphic to $Coclo(Y)$.*

Theorem *There is a mapping $\varphi \to \varphi^M$ of first order formulas of clones to those of monoids which satisfies the following condition: for any non-indiscrete topological space X and any closed formula φ of the theory of clones,*

$$Coclo(X) \models \varphi$$

if and only if

$$Mon(X) \models \varphi^M.$$

The universal minimal flow in the language of near ultrafilters

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We describe the universal minimal flow as a space of near ultrafilters that were introduced by Koçak and Strauss in []. We use this approach to identify universal minimal flows of groups of automorphisms of countably-homogeneous relational structures and to confirm Pestov's conjecture about the Ellis problem ([]) in case of discrete groups. This work unifies and extends results by Kechris, Pestov and Todorcevic in [] and Glasner and Gutman in [].

[9] The author was partially supported by GAUK 66509

On graph topologies and their counterparts in the realm of convenient topology

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As a contribution to the theory of "convenient topology" (founded by Gerhard Preuß , see []), the topological universe of multifilterspaces will be introduced. We define some suitable subcategories in order to link the classical uniform spaces (as defined by coverings - please notice the nice discussion []) into this. We define and observe a kind of hyperstructures and try to use some knowledge about this to prove precompactness of subsets of function-spaces. Furthermore, we use this hyperstructures to adopt the concept of graph topologies in function spaces for multifilterspaces, as developed for topological spaces by Naimpally ([]) and Poppe ([], []).

Weak extent,submetrizability and diagonal degrees

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H.W. Martin proved that separable spaces having a zero-set diagonal are submetrizable, while R.Z. Buzyakova proved that a space X having a zero-set diagonal and whose square has countable extent is submetrizable.

We give a simultaneous generalization of Martin and Buzyakova's result by proving that a space X having a zero-set diagonal and whose square has countable weak extent is submetrizable.

Finally, we provide some cardinality bounds involving various types of diagonal degree.

Strong homology group of continuous map

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Using the shape properties of continuous map normal cohomology functor from the category of maps to the category of long exact sequences of groups is constructed by V.Baladze ([], []). The main aim of this report is to study the strong homology of maps. It is proved that for each pro-chain map $f : C \longrightarrow C'$ there exists the long exact strong homological sequence:

$$... \longrightarrow H_m(C) \longrightarrow H_m(C') \longrightarrow H_m(f) \longrightarrow H_{m-1}(C) \longrightarrow ...$$

Using the obtained results strong homology functor $H_*(-)$ are constructed on the category M_{Top} of continuous maps of topological spaces. It is proved that the functor $H_*(-) : M_{T0p} \longrightarrow Ab$ satisfies the Boltianski type axioms. Besides, the isomorphism $H_*(f) \approxeq H_*(C_f)$ of strong homology group of continuous map $f : X \longrightarrow Y$ of topological spaces and strong homology group of mapping cone C_f of the map f is proved. As corollary, it is obtained that for each pair (X,A) of topological spaces there exists the long exact strong homological sequence:

$$... \longrightarrow H_m(A) \longrightarrow H_m(X) \longrightarrow H_m(C_i) \longrightarrow H_{m-1}(A) \longrightarrow ...,$$

where C_i is mapping cone of inclusion $i : A \longrightarrow X$. In the case when $i : A \longrightarrow X$ is cofibration and (X,A) is normally embedded pair there is the isomorphism $H_*(C_f) \approxeq H_*(X,A)$ of strong homology group of mapping cone C_i and strong homology group of pair (X,A).

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Group theoretic properties of the automorphism groups of minimal T_D topologies

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We will present group theoretic properties of the automorphism groups of minimal T_D topologies on a given fixed set X. This is achieved via the Stone duality between linear orders and minimal T_D topologies. In particular, given a dense linear order Ω, we generalize Cameron's construction of large free groups within Aut(Ω).

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On representation space and mappings

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Consider the equivalence relation $\sim$ on the power set of I^{ω} wich is defined by the notion of homeomorphism, i.e., two subspaces of the Hilbert cube are equivalent under $\sim$ if and only if they are homeomorphic. Then, by $C_{\sim}$ we mean the quotient set $\{C \subset I^{\omega}\}/\sim$, and by C is the set of all subcontinua of the Hilbert cube, up to homeomorphism. Let P a subset of C and α be a class of mappings having the composition property. Consider $X \in C$. Then we write$X \in \alpha - Cl(P)$ to mean that for each ϵ, there exists $Y_{\epsilon} \in P$ and $\epsilon-$map, $f_{\epsilon} \in \alpha$, from X onto Y_{ϵ}. The operator Cl induces a topology on C. In this talk we are going to give classes of subcontinua which either are open or closed sets on C regarding classes of mappings. And we will show the interior and the closure of other subsets of C.

Homogeneity in Non-regular Spaces

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It was shown in [] that the cardinality of any power homogeneous Hausdorff space X is at most $2^{L(X)t(X)pct(X)}$, where $pct(X)$ is the point-wise compactness type of X. Modifying techniques in [], we show that $L(X)$ in this bound can be replaced with $aL_c(X)$, the almost-Lindelöf degree of X with respect to closed sets. We also show the cardinality of a power homogeneous Hausdorff space X with finite Urysohn number is at most $d_\theta(X)^{\pi\chi(X)}$, where $d_\theta(X)$ is the θ-density of X. This represents a strengthening of a result in []. These bounds are equivalent to known bounds if the space X is regular. Additionally, we show that if X is an H-closed, Urysohn homogeneous space then X has the property that for every regular-closed subset A of X, $x \in A$, $y \notin A$, there exists a homeomorphism h of X such that $h(y) \in A$ and $h(x) \notin A$. This was previously shown to hold for compact homogeneous spaces X by Motorov, where A is any closed subset of X. Finally, we establish that the Katětov H-closed extension κX of any non-H-closed space X is never homogeneous, and the remainder $\sigma X \setminus X$ in the Fomin H-closed extension σX of any locally H-closed space is never power homogeneous.

Results about representation spaces

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Out of every class of homeomorphic continua we choose one member and we call such collection the representation space $\mathcal{C}$. For a subset $\mathcal{P} \subseteq \mathcal{C}$ we say that $X \in \mathrm{cl}\mathcal{P}$ if for every $\varepsilon > 0$ there is a space X_ε in $\mathcal{P}$ and a mapping $f_\varepsilon : X \to X_\varepsilon$ that is a surjective ε-mapping. This closure operator introduces a topology on the representation space. Some other variation of the definition can be considered if we require that the considered ε-mapping belongs to a given class of mappings like monotone, open or confluent.

In this talk we will review the results and problems concerning representation spaces.

Some aspects of topological dynamics on spaces with a free interval

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We study dynamics of continuous maps on compact metrizable spaces containing a free interval (i.e., an open subset homeomorphic to an open interval). A special attention is paid to relationships between topological transitivity, weak and strong topological mixing, dense periodicity, topological entropy and to the topological structure of minimal sets.

This is a joint work with Ľubomír Snoha and Vladimír Špitalský.

On normal functors

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A well-known Katětov [] theorem states, that the hereditary normality of X^3 for compact X implies the metrizability of X. V.V. Fedorchuk [] generalized the Katětov theorem for any normal functor of degree ≥ 3.

In this connection we generalize the definition of normal functor to a category of paracompact p-spaces and perfect mappings and prove the following

Theorem *Let X be paracompact p-space, $\mathcal{F}$ — normal functor of degree ≥ 3 in category $\mathcal{P}$. Then if the space $\mathcal{F}(X)$ is hereditarily normal, X is a metrizable space.*

Productively sequential spaces

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It is well known that $\xi\times\tau$ is (strongly) Fréchet for each space of countable character τ if and only if ξ is *strongly Fréchet*. In 2004 it was shown by F. Jordan and F. Mynard that $\xi \times \tau$ is (strongly) Fréchet for each strongly Fréchet space τ if and only if ξ is *productively Fréchet*. The class of *strongly sequential* spaces ξ, that is, such that $\xi\times\tau$ is *sequential* for each space of countable character τ, was characterized structurally and internally by F. Mynard in 2000, providing a complete answer to a problem posed by Y. Tanaka of 1976.

Here we characterize the class of *productively sequential* spaces ξ, that is, such that $\xi \times \tau$ is *sequential* for each strongly sequential space τ.

A general approach to such quests hinges on the fact that numerous classes of convergence spaces θ can be characterized with the the aid of functorial inequalities of the type $\theta \geq JV\theta$, where J is a reflector and V a coreflector. This fact has enabled us to use, in all such situations, a fundamental equivalence (due to F. Mynard) between $\theta\times JV\tau \geq L\,(\xi\times\tau)$ for each space τ, and $\theta \geq \mathrm{Epi}^{L}_{JV}\,\xi$, where $\mathrm{Epi}^{L}_{JV}\,\xi$ is the *JV-modified L-bidual* of ξ.

Canonizing Borel equivalence relations for the Silver ideal

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Let X be a Polish space, $I \subseteq \mathcal{P}(\mathcal{X})$ a σ-ideal on it and $E \subseteq X \times X$ a Borel (analytic) equivalence relation. We say that E is in the spectrum of I if there exists a Borel set $B \in I^+$ such that $\forall C \in (I^+ \cap \mathrm{Borel(B)})$ $E \restriction C$ is Borel bireducible with $E \restriction X$. Otherwise, E can be canonized to some equivalence relation of less complexity for the ideal I (see the forthcoming book []).

Here we focus on the Silver ideal and the equivalence relations given by analytic P-ideals. The relation E_0 on 2^ω, where xE_0y if $x \bigtriangleup y \in \mathrm{Fin}$, is an example of a relation which lies in the spectrum of Silver. The main result is:

Theorem *Let $\mathcal{I}$ be an analytic P-ideal, B a positive Borel (analytic) set for the Silver ideal and $E \subseteq B^2$ an equivalence relation Borel reducible either to $E_\mathcal{I}$ on 2^ω or to the equivalence relation ℓ^p on $\mathbb{R}^\omega$ for $p \in \mathbb{N}$. Then there exists a positive Borel subset $C \subseteq B$ such that $E \restriction C$ is C^2 or a subset of $E_0 \restriction C$.*

Since we can expect a subequivalence of E_0 as a result we then attempt to classificate subequivalences of E_0 which are mutually different by the means of the Silver ideal.

On the Gurarii space

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The Gurarii space $\mathbb{G}$ is the unique separable Banach space of almost universal disposition for finite-dimensional spaces. This means that, given any finite-dimensional Banach spaces $X \subset Y$, given $\varepsilon > 0$, every isometric embedding $f : X \to \mathbb{G}$ extends to an ε-isometric embedding $g : Y \to \mathbb{G}$. This space was found by Gurarii in 1966. Its uniqueness up to isometry was proved by Lusky in 1976.

We shall present a category-theoretic description of the Gurarii space, obtaining simpler proofs of some known as well as some new properties of this space. We shall also discuss universal projections on the Gurarii space.

[12] PHD fellowships important for regional development.

Topologies generated by weak selection topologies

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Given an infinite X, a weak selection is a function $f : [X]^2 \to X$ such that $f(\{x,y\}) \in \{x,y\}$ for each $\{x,y\} \in [X]^2$. A weak selection f on X defines a relation $x \prec_f y$ if $f(\{x,y\}) = x$ whenever $x,y \in X$ are distinct. The topology τ_f on X generated by the weak selection f is the one which has the family of all intervals $(\leftarrow, x) = \{y \in X : y \prec_f x\}$ and $(x, \rightarrow) = \{y \in X : x \prec_f y\}$ as a subbase. The paper deals with the topological spaces (X,τ) for which τ is the supremun of a family of topologies defined by a weak selections on X.

Whitney Levels on Hyperspaces of Non Metrizable Continua

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A Hausdorff continuum is a nonempty, compact, connected Hausdorff space. In this talk I will give definitions of (a) Whitney Levels and (b) Ordered Arcs for Hausdorff Continua, which generalize the definition of the metric case.

If the continuum is metric it is known that every subcontinuum belongs to a Whitney Level. In this talk I will present examples of non metrizable continua where no one of its subcontinua belongs to a Whitney Level.

Base dimension-like functions of the type ind

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In [] base dimension-like functions of the type ind were introduced. These functions were studied only with respect to the property of universality. Here, we study these functions with respect to other standard properties of dimension theory.

Entropy on abelian groups

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A notion of algebraic entropy for automorphisms of abelian groups was introduced by Peters. We modify appropriately this definition to extend it to all endomorphisms of abelian groups.

First, we prove the Algebraic Yuzvinski Formula showing that the value of the algebraic entropy of an endomorphism $\phi : \mathbb{Q}^n \to \mathbb{Q}^n$ equals the Mahler measure of its characteristic polynomial $p_\phi(X) = sX^n + a_1X^{n-1} + \ldots + a_n$ over $\mathbb{Z}$, that is, $h(\phi) = \log s + \sum_{|\lambda_i|>1} \log |\lambda_i|$, where $\{\lambda_i : i = 1, \ldots, n\}$ are the eigenvalues of ϕ. This is the algebraic counterpart of the formula proved by Yuzvinski for the topological entropy $h_{top}(-)$ of solenoidal automorphisms.

Then, we give several fundamental properties of the algebraic entropy as applications of the Algebraic Yuzvinski Formula, starting from the Addition Theorem and the Uniqueness Theorem. Moreover, we deduce an extension of both Weiss Bridge Theorem and Peters Bridge Theorem about the connection between the algebraic entropy of an endomorphism $\phi : G \to G$ and the topological entropy of its Pontryagin dual $\widehat{\phi} : G \to G$, showing that $h(\phi) = h_{top}(\widehat{\phi})$. A last relevant application concerns the growth of the trajectories of ϕ in G.

Continuity of convolution of test functions on Lie groups

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Let G be a finite-dimensional Lie group, with Haar measure μ, and

$$\beta : C_c^\infty(G) \times C_c^\infty(G) \to C_c^\infty(G), \quad \beta(f,g) := f * g$$

with $(f * g)(x) := \int_G f(y)g(y^{-1}x)\, d\mu(y)$ be the convolution map. The following results will be presented and explained:

1. If G is σ-compact, then β is continuous.

2. If G is not σ-compact, then β is not continuous.

This is joint work in progress with Lidia Birth (Paderborn).

On Stone space of one boolean algebra

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We regard a compactification B_2N_2 of a countable discrete space N_2. This compactification, as well as Bell's compactification BN, is Stone space of some Boolean algebra of subsets of N_2.

$N_2 = \{f|_n : f \in P_2, n \in \omega\}$, where

$P_2 = \{f \in \omega^\omega : 0 \leqslant f(n) \leqslant 1 \text{ for all } n \in \omega\}$

We order N_2: $s \leqslant t$ if t is an extension of s for $s, t \in N_2$.

We prove that:

1. $N_2 \subset BN$ and $\overline{N_2}^{BN}$ is homeomorphic to B_2N_2.

2. The limits of infinite chains are isolated points in $B_2N_2 \setminus N_2$.

3. If $A \subseteq N_2$ is an infinite strict anti-chain, then $\overline{A} \setminus A$ is a clopen in $B_2N_2 \setminus N_2$. Here we call an anti-chain $A \subseteq N_2$ a strict anti-chain, if $\operatorname{dom} x \neq \operatorname{dom} y$ for all $x, y \in A$, $x \neq y$.

Theorem *If $x \in B_2N_2 \setminus N_2$ is a non-isolated point, then for every neighborhood O_x there is a infinite strict anti-chain $A \subseteq N_2$ such that $\overline{A} \subseteq O_x$, therefore $c(O_x) = 2^\omega$.*

Theorem *If the closure $\overline{A}$ of a set $A \subseteq N_2$ is a copy of βN and $\overline{A} \setminus A$ is a clopen in $B_2N_2 \setminus N_2$, then A is a union of finitely many strict anti-chains.*

Example *There is $A \subseteq N_2$ such that $\overline{A}$ is a copy of βN and $\overline{A} \setminus A$ is not a clopen in $B_2N_2 \setminus N_2$.*

Example *There is $A \in B_2$ (therefore $\overline{A}$ is clopen in B_2N_2) such that $\overline{A}$ is not a copy of βN and there are no isolated points in $\overline{A} \setminus A$.*

On converging sequences and copies of βN in one compactification of N

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We consider a compactification BN of a counatable discrete space N with ccc non-separable remainder. This compactification was constructed by M. Bell as Stone space of some Boolean algebra of subsets of $N = \{f|_n : f \in P,\ n \in \omega\}$, where

$$P = \{f \in \omega^\omega : 0 \leqslant f(n) \leqslant n + 1 \text{ for all } n \in \omega\}.$$

We order N: $s \leqslant t$ if t is an extension of s for s, $t \in N$.

We examine closures $\overline{A}$ of some subsets $A \subseteq N$. We proved earlier that if $A \subseteq N$ is an infinite chain, then $|\overline{A} \setminus A| = 1$, i. e. A is a converging sequence, and if $A \subseteq N$ is a strict anti-chain, then $\overline{A}$ is a copy of βN. Here we call an anti-chain $A \in N$ a strict anti-chain, if $\operatorname{dom} x \neq \operatorname{dom} y$ for all x, $y \in A$, $x \neq y$.

Theorem *If a set $A \subseteq N$ is such that $|\overline{A} \setminus A| = 1$, then there is a finite set $K \subseteq A$ such that $A \setminus K$ is a chain.*

Theorem *If the closure $\overline{A}$ of a set $A \subseteq N$ is a copy of βN, then A is a union of finitely many anti-chains.*

Note, that there are two anti-chains A_1, $A_2 \subseteq N$ such that $\overline{A_1}$ and $\overline{A_2}$ are copies of βN, but $\overline{A_1 \cup A_2}$ is not a copy of βN.

Dense subspaces vs closure-preserving covers of function spaces

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We study when a space $C_p(X)$ has a closure-preserving cover $\mathcal{C}$ such that every element of $\mathcal{C}$ has a property $\mathcal{P}$. We establish that for many properties $\mathcal{P}$ this implies that $C_p(X)$ has a dense subspace that has $\mathcal{P}$. In particular, if $\mathcal{P}$ belongs to the following list: Lindelöf property, Lindelöf Σ-property, countable network weight, countable extent, σ-compactness, K-analyticity, analyticity, stability, and $C_p(X)$ has a closure-preserving cover such that every $C \in \mathcal{C}$ has $\mathcal{P}$ then $C_p(X)$ has a dense subspace with the property $\mathcal{P}$.

It is also proved that if $\mathcal{P}$ is a hereditary property and $C_p(X)$ is the union of a closure-preserving family $\mathcal{C}$ such that every $C \in \mathcal{C}$ is closed in $C_p(X)$ and has $\mathcal{P}$ then $C_p(X)$ also has $\mathcal{P}$. If $\mathcal{P}$ is either closed-hereditary or preserved by quotient images and $C_p(X,[0,1])$ is the union of a closure-preserving family $\mathcal{C}$ such that every $C \in \mathcal{C}$ is closed in $C_p(X,[0,1])$ and has $\mathcal{P}$ then $C_p(X,[0,1])$ must have $\mathcal{P}$.

ω^*, ω_1^* and non-trivial autohomeomorphisms

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The Katowice Problem asks: are ω^* and ω_1^* homeomorphic?

Efforts to prove that the answer is negative have produced consequences of a positive answer; the hope being that the conjunction of these consequences would be equivalent to $0 = 1$. We add one more consequence to the list:

Theorem *If ω^* and ω_1^* are homeomorphic then there is a non-trivial autohomeomorphism of ω^*.*

Classifying spaces of remote points for some metrizable spaces

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For a Tychonoff space X, a remote point of βX is a point $p \in X^*$ such that for every nowhere dense subset A of X we have $p \notin cl_{\beta X}(A)$. Let $\rho(X)$ denote the subspace of remote points of βX. Our focus is the following problem: \`Let X be a metrizable space. Characterize those (metrizable) Y such that $\rho(X)$ is homeomorphic to $\rho(Y)$.' In [], Woods gives an answer to the problem when both X and Y are locally compact and non-compact metrizable spaces. In our talk, we present results for the case when X is completely metrizable.

[13] The first author was supported by CONACyT scholarship for Doctoral Students.

A metrization theorem for small Frechet groups

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We prove that it is consistent with the negation of the Continuum hypothesis that every separable Frechet group of weight less than continuum is metrizable.

Sharkovskii's theorem for random dynamical systems

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Topological methods of random dynamical systems have received a lot of interest recently. As an example, we present Sharkovskii's-type results for random dynamical systems. In particular, we give a partial solution of the Klunger's conjecture on a structure of periodic orbits of random subshifts of finite type. We also describe the dynamics generated by higher order random difference equations.

Strong sequences in partially ordered sets

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The strong sequences method was introduced by B. A. Efimov, as a useful method for proving famous theorems in dyadic spaces: Marczewski theorem on cellularity, Shanin theorem on a calibre, Esenin-Volpin theorem and Erdös-Rado theorem. The aim of this paper is to introduce a new cardinal invariant *s*- a length of the strong sequence and to investigate relations between *s* and other well known invariants like: saturation, boundeness, cofinality, calibre.

Preservation of the Lindelöf property and infinite games on posets

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For a topological space (X,τ) and a forcing notion $\mathbb{P}$, let $\tau^{\mathbb{P}}$ denote a $\mathbb{P}$-name for the topology on $\check{X}$ generated by $\check{\tau}$ in a generic extension by $\mathbb{P}$. We say $\mathbb{P}$ preserves a certain topological property of (X,τ) if $(\check{X}, \tau^{\mathbb{P}})$ is forced to have the same property.

In this talk I will present several results about preservation of topological covering properties under forcing extensions, including the following theorem. A cut-and-choose game $\mathrm{CG}^{<\alpha}(1)$ on a poset $\mathbb{P}$ is played by two player One and Two for α innings. In the beginning One picks $p \in \mathbb{P}$. In each inning ξ, One gives a maximal antichain W_ξ in $\mathbb{P}$ below p, and Two picks an element b_ξ from W_ξ. Two wins if, for any $\gamma < \alpha$, $\{b_\xi : \xi < \gamma\}$ has a lower bound in $\mathbb{P}$.

Theorem *For a poset $\mathbb{P}$, if* One *has no winning strategy in the game $CG^{<\omega_1}(1)$ on $\mathbb{P}$, then $\mathbb{P}$ preserves hereditary Lindelöfness.*

In the above theorem we cannot replace the assumption on $\mathbb{P}$ by "One does not have a winning strategy in the game $\mathrm{CG}^{\omega}(1)$ on $\mathbb{P}$," where $\mathrm{CG}^{\omega}(1)$ stands for $\mathrm{CG}^{<(\omega+1)}(1)$.

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Approximate fixed point nets and sequences

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Let X be a Hausdorff topological vector space, $C \subset X$ a nonempty closed convex bounded set and $f : C \to C$ be a mapping enjoying some continuity property. I will discuss the question of existence of a net or sequence (x_i) in C such that $x_i - f(x_i)$ converge in some sense to zero. Such a net exists under very weak assumptions. Existence of such a sequence in case of metrizable locally convex spaces can be characterized via non-containment of an ℓ_1-sequence. These results are contained in the papers [] and [].

The De Groot Dual Revisited

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Almost precisely ten years ago, at TOPOSYM 2001 there was announced a partial solution of a question of M. Mislove and J. Lawson, stated in Open Problems in Topology I as Problem 540. After ten years, it is a good occasion to recapitulate the status.

Let (X,τ) be a topological space. Recall that a topology τ^d is called *co-compact* or *de Groot dual* of τ, if it is generated by the family of all compact saturated sets used as its closed base. The original question of J. Lawson and M. Mislove, motivated by their research in domain theory, was stated as follows:

Question *Characterize those topologies that arise as dual topologies.*

Question *If one continues the process of taking duals, does the process terminate after finitely many steps with topologies that are dual of each other?*

The author's result communicated at TOPOSYM ten years ago yielded the answer "Yes" for the second question, while the more difficult first question has remained open. In the current contribution we will discuss some natural related questions and further developments in the topic.

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A dichotomy for the convex spaces of probability measures

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We show that every nonempty compact and convex space M of probability Radon measures either contains a measure which has "small" local character in M or else M contains a measure of "large" Maharam type. Such a dichotomy is related to several results on Radon measures on compact spaces and to some properties of Banach spaces of continuous functions.

Small retractions on Peano curves

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S. Mazurkiewicz proved in 1933 that any Peano curve X can be mapped onto graphs in X by arbitrary small continuous self-maps. The purpose of this note is to present a natural and short proof of a stronger theorem, based on the Bing's Brick Partitioning Theorem. It is quite possible that the theorem was known but I could not find any reference.

Theorem *If X is a Peano curve, then for every $\epsilon > 0$ there exist a connected graph $G \subset X$ and a continuous ϵ-retraction of X onto G.*

On inductive dimension modulo a simplicial complex

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For a given simplicial complex K, V. V. Fedorchuk has recently introduced the dimensions K-dim and K-Ind of normal spaces. If K consists of two points (without any edge), then $K\text{-dim} = \dim$ and $K\text{-Ind} = \text{Ind}$. Fedorchuk has proved that $K\text{-dim} \leq K\text{-Ind}$ for all normal spaces, and $K\text{-dim} = K\text{-Ind}$ for metrizable spaces. Moreover, for every integer $n \geq 2$ and simplicial complex K with a non-contractible join $K * K$, he has constructed a separable, first countable compact space X_n such that $K\text{-dim}\, X_n = n < 2n - 1 \leq K\text{-Ind}\, X_n \leq 2n$.

Let K be a non-contractible simplicial complex. We consider a transfinite extension of K-Ind.

Problem *Let n be a natural number, α an ordinal, and $\alpha \geq n \geq 1$. Under what circumstances are there compact spaces with* $K\text{-Ind} = \alpha$ *and* $K\text{-dim} = n$*? Can all (connected) components of such a space be metrizable?*

In the presentation we shall briefly sketch a construction, and state an exhaustive answer to the first question above. To the second one, we have an answer in a very particular case.

Retracts of universal homogeneous structures

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We present a category-theoretic characterization of retracts of Fraïssé--Jónsson limits. It turns out that these are precisely the injective objects with respect to the given Fraïssé-Jónsson class (category). Our characterization extends a recent result of Dolinka [] from model theory.

As a sample application, we characterize non-expansive retracts of Urysohn's universal metric space $\mathbb{U}$. Namely, a Polish space (X,d) is a non-expansive retract of $\mathbb{U}$ if and only if it is *finitely hyperconvex*, i.e. for every finite family of closed balls

$$\mathcal{B} = \{\mathcal{B}(x_0,r_0),\ldots,\mathcal{B}(x_{n-1},r_{n-1})\}$$

with $\bigcap \mathcal{B} = \emptyset$, there exist $i,j < n$ such that $d(x_i,x_j) > r_i + r_j$.

Some applications of tiny sequences

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Recall open-open game introduced in [] called G. The lenght of the game is ω. Two players I and II take turns playing. At the n-th move I Player chooses a non-empty open subset $A_0 \subseteq X$ at the beginning. Then Player II chooses a non-empty open subset $B_0 \subseteq A_0$. Player I chooses a non-empty open subset $A_n \subseteq X$ at the n-th inning, and then Player II chooses a non-empty open subset $B_n \subseteq A_n$. Player I wins, whenever the union $B_0 \cup B_1 \cup \ldots \subseteq X$ is dense, otherwise Player II wins.

A game G_7 is played as follows: II and I play an inning per positive integer. In the n-th inning II chooses $\mathcal{O}_\mathrm{n}$, a maximal family of pairwise disjoint open sets. I responds with T_n, a finite subset of $\mathcal{O}_\mathrm{n}$. A play $\mathcal{O}_1,\mathcal{T}_1,\ldots,\mathcal{O}_\mathrm{n},\mathcal{T}_\mathrm{n},\ldots$ of G_7 is won by I if $\bigcup_{n\in\omega} T_n$ is dense subset of X; otherwise, II wins.

We examine the class of II-favorable spaces i.e. II player has winning strategy in the open-open game G. A space Y is *universally Kuratowski-Ulam** (for short, *uK-U* space*), whenever for a topological space X and a nowhere dense set $E \subseteq X \times Y$ the set

$$\{x \in X : \{y \in Y : (x,y) \in E\} \text{ is not nowhere dense in } Y\}$$

is meager in X ([]). I and Sz. Plewik ([]) shown that a compact I-favorable space is universally Kuratowski-Ulam and posed a question:

Question *Does there exist a compact universally Kuratowski-Ulam space which is not I-favorable?*

[16] The author was partially supported by ESF

We will partial answer on this question namly we prove that II-favorable space is not universally Kuratowski-Ulam We show that game G and G_7 are not equivalent.

The Katětov construction revisited

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We study Katětov's construction (see []) modified for a T_0-quasi-metric space. Our approach is related to the recent work of the first author with Kemajou and Otafudu about a concept of hyperconvexity in such spaces.

[17] The first author was partially supported by the National Research Foundation of South Africa

Improved nearness research IV

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In some previous papers generalized nearness was considered in connexion with unifications and topological extensions, respectively. The last part of series deals with those supernear spaces, which cover closely LODATO spaces, b-supertopologies and hence EF-proximities as well. Moreover, we will show that some special kind of these spaces have a local topological extension iff they are superbunch spaces. Consequently, this fundamental result leading us to a further generalization of LODATO's Theorem, and in addition Doîtchinov's achievement can be dealt with, but in a modified fashion.

Vilenkin duality revisited: A unifying duality for abelian groups

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For an abelian topological group G, let G^* denote the group of continuous characters of G. The *Pontryagin dual* $\hat{G}$ of G is the group G^* equipped with the compact-open topology. Although the evaluation map $\alpha_G : G \to \hat{\hat{G}}$ is a topological isomorphism for every locally compact G, in general, α_G need not even be continuous.

In 1951, Vilenkin introduced topobornological groups (groups with "boundedness"), which are topological groups that also carry a bornology that is compatible with the operations. He defined the dual G^* of a topobornological group G to be the group G^* equipped with the bounded-open topology, and the equicontinuous bornology. Vilenkin showed that the assignment $\mathrm{G} \to \mathrm{G}^*$ is "well behaved", and in particular, the evaluation $\eta_{\mathrm{G}} : \mathrm{G} \to \mathrm{G}^{**}$ is always continuous and bounded.

The aim of this talk is to demonstrate that the Vilenkin duality is the "right" framework for studying properties and problems related to the Pontryagin duality, the Comfort-Ross duality, and the Bohr-compactification. Two new constructions for topobornological groups (tensor product and internal hom) will also be presented.

[18] The author gratefully acknowledges the generous financial support received from NSERC

On freely decomposable maps

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Freely decomposable and strongly decomposable maps were introduced by G. R. Gordh and C. B. Hughes as a generalization of monotone maps with the property that these maps preserve local connectedness under inverse limits. We study further these types of maps, generalize some of their results and present examples showing that no further generalization is possible.

Epireflective subcategories of T_2Unif, $Unif$, and $CompT_2Ab$, closed under epimorphic images

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Each subcategory of a category is considered full and isomorphism closed, and is identified with its class of objects.

D. Petz proved: the only non-trivial, epireflective (closed under products and closed subspaces) subcategory of T_2, closed under epimorphic (dense) images, is that of compact Hausdorff spaces.

The non-trivial subcategories of T_2Unif with the same properties are the compact T_2 spaces, and, for any infinite cardinal α, T_2 uniform spaces having a base of uniform coverings of cardinalities$<\alpha$.

For *Unif* (then closedness under products, subspaces and onto images) we have an analogous theorem: compact T_2 spaces are replaced by indiscrete spaces.

For compact T_2 Abelian groups the non-trivial above subcategories (closed under products, closed subgroups and onto images) are those, whose elements X have neighbourhood bases of 0, consisting of open subgroups X_α, such that X/X_α is finite cyclic, of order in a subset of N, closed under divisors and least common multiples.

The talk is based on [] and [].

[19] Partially supported by OTKA grants 68398, 75016, 81146

Trees, ultrametric spaces, and their automorphism groups

Maciej Malicki

We would like to present a number of results about automorphism groups of countable rooted trees, and isometry groups of Polish ultrametric spaces, equipped with naturally defined Polish topologies.

We say that a group G has uncountable strong cofinality if whenever G is a union of a countable chain of subsets $A_0 \subset A_1 \subset \ldots$, then $A_m^k = G$ for some $k,m \in N$. A Polish topological group G has ample generics if the diagonal action of G on G^n by conjugation has a comeagre orbit for every $n \in N$.

Let T be a countable rooted tree, $G = \mathrm{Aut}(T)$, and, for $X \subseteq T$, let $\mathrm{ACL}_T(X)$ denote the algebraic closure of X in T. We have

1. G has uncountable strong cofinality iff $\mathrm{ACL}_T(\emptyset)$ is finite;
2. G has an open subgroup with ample generics iff $\mathrm{ACL}_T(X)$ is finite for every finite $X \subseteq T$.

Then we turn to Polish ultrametric spaces. Among other results, we give an explicit and inner characterization of isometry groups of a large class of such spaces in terms of a 'separable' variant of the unrestricted generalized wreath product. Combining this with the results on rooted trees, we characterize those ultrametric spaces, whose isometry groups have uncountable strong cofinality.

From this point of view, ultrametric spaces can be viewed as highly homogeneous structures (even in cases when the isometry group is not transitive.) On the other hand, we introduce a canonical operation of forming a new ultrametric space out of a given one that reduces every Polish ultrametric space to a rigid space.

Buried Points in Julia Sets

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The set of buried points J' of the Julia set J of a rational function (also called the residual Julia set) is the set of points of the Julia set not on the boundary of any complementary domain of J. By well-known dynamical properties of Julia sets, J' is a fully invariant dense G_δ subset of J or empty. Mayer and Curry have conjectured that there are only a few topological types of buried points of rational Julia sets. We report on a continuing investigation that suggests on the contrary that there are very many topological types. We focus on the buried points of the Julia sets of singularly perturbed families such as $z \to z^n + \frac{\lambda}{z^n}$ and $z \to z^n + c + \frac{\lambda}{z^n}$.

On topologies on X as points within $2^{\mathcal{P}(X)}$

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Given a nonempty set X, Bankston defines a *superfamily* over X as a collection of subsets of $\mathcal{P}(X)$. A topology on X specifies a natural subset of $\mathcal{P}(X)$ and thus the lattice Top(X) of all topologies on X is a superfamily over X. We present some topological properties of superfamilies over X by identifying $\mathcal{P}(\mathcal{P}(X))$ with the totally disconnected compact Hausdorff space $2^{\mathcal{P}(X)}$. In particular, we investigate topological properties of Top(X) as a superfamily and give sufficient model-theoretic conditions for a superfamily to be compact.

[20] The second author was supported by IRCSET

Base dimension-like functions of the type ind

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In [] base dimension-like functions of the type ind were introduced. These functions were studied only with respect to the property of universality. Here, we study these functions with respect to other standard properties of dimension theory.

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[22] Work supported by the Caratheodory Programme of the University of Patras.

Hausdorff ultrafilters and Katětov order

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Hausdorff ultrafilters have been studied probably since ultraproducts were defined. Recently, M. di Nasso and M. Forti reintroduced this notion when they were studing the S topolgy for non-standard model of arithmetics. The S-topology is the generated by the sets ${}^*A$ for $A \subseteq \mathbb{N}$. An ultrafilter $\mathcal{U}$ is *Hausdorff* if the topology S is Hausdorff on the ultrapower $\prod_U \mathbb{N}$. This condition has several combinatorial equivalences. I have found that the F_σ-ideal $\mathcal{G}_{\mathrm{fc}}$ of all the finitely chromatic subsets of $[\mathbb{N}]^2$ is critical for Hausdorff ultrafilters in the Katětov order, in other words, the Hausdorff ultrafilters are exactly the $\mathcal{G}_{\mathrm{fc}}$-ultrafilters (in the sense of Baumgartner). To know the position of $\mathcal{G}_{\mathrm{fc}}$ in the Katětov order enable us to show some relations of the Hausdorff ultrafilters with other classes of ultrafilters, including those that satisfy a Fubini-type property.

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On the complexity of compact 3-manifolds

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We deal with Matveev complexity of compact orientable 3-manifolds represented via Heegaard diagrams. The definition of modified Heegaard complexity of Heegaard diagrams and manifolds is given, and a comparison with Matveev complexity is shown. As a relevant example, a class of manifolds which are generalizations of Dunwoody manifolds (including cyclic branched coverings of two-bridge knots and links, torus knots, some pretzel knots, and some theta-graphs) is introduced. Upper bounds for their Matveev complexity, which linearly depend on the order of the covering, are obtained, via modified Heegaard complexity. Also some lower bounds are provided, using homology arguments.

Diagonality in spaces of open sets

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Let X be a topological space and let $\mathcal{O}_X$ be the set of open subsets. It is well known that the Scott convergence on the complete lattice $(\mathcal{O}_X, \subseteq)$ given by

$$U \in \lim \mathcal{F} \Leftrightarrow U \subseteq \bigcup_{F \in \mathcal{F}} \mathrm{int}(\bigcap_{O \in F} O),$$

is topological if and only if it is pretopological, if and only if X is core compact. Note that it homeomorphic to the upper Kuratowski convergence on closed subsets of X.

As topologies are exactly the diagonal pretopologies, it raises the question whether the Scott convergence (homeomorphically upper Kuratowski convergence) is always diagonal. We characterize diagonality of the above convergence and show that it is not always diagonal, and can be diagonal without being pretopological.

Further, Scott convergence on $\mathcal{O}_X$ can be defined for a general convergence space X, but the formula above has to be altered. In this case, we show that Scott convergence can be pretopological but not diagonal.

If time permits, core compactness of the Scott convergence and Scott topology on $\mathcal{O}_X$ will also be discussed.

On closed frame homomorphisms

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Given a homomorphism $h : L \to M$ between completely regular frames, there is a unique extension $h^{\gamma} : \gamma L \to \gamma M$ for $\gamma \in \{\beta,\lambda,\upsilon\}$ where βL is the Stone- compactification, λL the Lindelöf coreflection and υL the realcompact coreflection. It is known that any frame homomorphism into a compact regular frame is closed. So the Stone-lift $h^{\beta} : \beta L \to \beta M$ of any frame homomorphism $h : L \to M$ is closed regardless of whether h is closed or not. We investigate conditions which ensure that the Lindelöf coreflection $h^{\lambda} : \lambda L \to \lambda M$ and the realcompact coreflection $h^{\upsilon} : \upsilon L \to \upsilon M$ are closed.

Universal valued Abelian groups

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Denote by $\mathfrak{G}$ the class of all separable valued Abelian groups. Let $\mathfrak{G}_1(0)$ stand for the class of all members of $\mathfrak{G}$ with diameter no greater than 1. For natural $N > 1$ let $\mathfrak{G}_\infty(\mathfrak{N})$ consist of all groups $(G, +, p) \in \mathfrak{G}$ of exponent N, and let $\mathfrak{G}_1(\mathfrak{N}) = \mathfrak{G}_1(0) \cap \mathfrak{G}_\infty(\mathfrak{N})$. Finally, let $\mathfrak{G}_\infty(0)$ be the class of all groups $(G, +, p) \in \mathfrak{G}$ such that $\lim_{n\to\infty} p(ng)/n = 0$ for each $g \in G$.

Fix $r \in \{1, \infty\}$ and $N \in \{0, 2, 3, 4, \ldots\}$. The following three results will be discussed.

Theorem *There is a unique (up to isometric group isomorphism) valued Abelian group, denoted by $\mathbb{G}_{\mathrm{r}}(\mathbb{N})$, with the following three properties.*

1. *$\mathbb{G}_{\mathrm{r}}(\mathbb{N})$ is complete and $\mathbb{G}_{\mathrm{r}}(\mathbb{N}) \in \mathfrak{G}_{\mathfrak{r}}(\mathfrak{N})$; if $N = 0$, finite rank elements of $\mathbb{G}_{\mathrm{r}}(\mathbb{N})$ form a dense subset of $\mathbb{G}_{\mathrm{r}}(\mathbb{N})$,*

2. *Let $(H, +, q) \in \mathfrak{G}_{\mathfrak{r}}(\mathfrak{N})$, K be a closed subgroup of H and $\varphi : K \to \mathbb{G}_{\mathrm{r}}(\mathbb{N})$ be a continuous homomorphism whose range has compact closure in $\mathbb{G}_{\mathrm{r}}(\mathbb{N})$. There is a continuous homomorphism $\psi : H \to \mathbb{G}_{\mathrm{r}}(\mathbb{N})$ extending φ such that $\ker \psi = \ker \varphi$. If φ is isometric, so is ψ.*

3. *Every (topological or isometric) isomorphism between two compact subgroups of $\mathbb{G}_{\mathrm{r}}(\mathbb{N})$ is extendable to an automorphism of the (topological or valued) group $\mathbb{G}_{\mathrm{r}}(\mathbb{N})$.*

In particular, $\mathbb{G}_{\mathrm{r}}(\mathbb{N})$ is universal for $\mathfrak{G}_{\mathfrak{r}}(\mathfrak{N})$.

Theorem *$\mathbb{G}_{\mathrm{r}}(\mathbb{N})$ is homeomorphic to a Hilbert space. $\mathbb{G}_{\mathrm{r}}(\mathbb{N})$ is (metrically) universal for separable metric spaces. $\mathbb{G}_{\mathrm{r}}(\mathbb{N})$ is Urysohn as a metric space iff $N \in \{0, 2\}$.*

Theorem *Every group of class* $\mathfrak{G}_\mathrm{r}(\mathfrak{N})$ *with a `continuous linear-like' structure may be enlarged to* $\mathbb{G}_\mathrm{r}(\mathbb{N})$ *with a `continuous linear-like' structure.*

Question *Does there exist a universal (for certain class) `continuous linear-like' structure on* $\mathbb{G}_\mathrm{r}(\mathbb{N})$ *?*

On supercovering spaces

PD Dr. Dieter Leseberg, Dr. S. B. Nimse, Zohreh Vaziry*

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It is well known that uniform spaces are a nice framework for studying metric properties. In the past two equivalent concepts were defined by A. Weil in 1937, named *diagonal uniformities* and by J. W. Tukey in 1940, called *covering structures*. In 1974 Herrlich generalized both to *nearness* respectively *uniform cover spaces* in order to get a common generalization of uniformity, proximity and topology. Hence, completeness and completion were intensely studied for the corresponding spaces.

In our paper we have found a natural generalization of uniform cover spaces and supertopologies as defined by Doitchinov in 1985. In this broader realm of so-called *supercovering spaces* we extend the term of completeness and construct a completion of symmetric graded U_1 -supercovering spaces. Then its completion is still epireflective in the former one and offer us in a special case the simple completion of a separated N_1- space in the sense of Herrlich.

A convergent sequence which is not an IFS-attractor.

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We deal with the part of Fractal Theory related to finite families of contractions called iterated function systems (IFS). An *attractor* is a compact set, invariant for such a family $\mathcal{F}$. In other words, an IFS-attractor is the unique fixed-point of the natural contraction (induced by $\mathcal{F}$) acting on the hyperspace of non-empty compact sets endowed with the Hausdorff metric. It is easy and well known that the standard Cantor set is an IFS-attractor. On the other hand, it can be topologically embedded in the real line so that its image is not an IFS-attractor.

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On weak mixing and multi-transitivity

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Let $f : X \to X$ be a continuous map on a compact metric space. We will say that f is: *multi-transitive* if for each $m \in \mathbb{N}$ the map $f \times f^2 \times \ldots \times f^m$ is topologically transitive; *Δ-transitive* if for each $m \in \mathbb{N}$ there is a residual set $Y \subset X$ such that for every point $x \in Y$ the tuple $(x, \ldots, x) \in X^m$ has a dense orbit in X^m under the map $f \times f^2 \times \ldots \times f^m$.

In this talk we will survey some results on the above properties. In particular we will provide partial answers to the following questions:

Question *Does mulit-transitivity imply Δ-transitivity?*

Question *Is multi-transitivity somehow related to topological weak mixing (i.e. transitivity of $f \times f$)?*

The Hyperspace of Meager Subcontinua

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Given a continuum X, we define the following hyperspaces:

$$C(X) = \{A \in X : A \text{ is connected, closed and nonempty}\},$$

The Hyperspace of Subcontinua of X,

$$M(X) = \{A \in \mathrm{C}(X) : int(A) = \emptyset\},$$

The hyperspace of Meager Subcontinua of X, and for $x \in X$:

$$Mcs(x) = \bigcup \{A \in M(X) : x \in A\}.$$

The Meager Composant of x.

In this talk, we discuss some general topological properties of the hyperspace $M(X)$. We relate properties of $M(X)$ with properties of the continuum X. Also we characterize the subcontinua of some special families of continua by using the structure of their hyperspace M(X).

Finally we show some general properties of $Msc(x)$ and we answer a question stated by David P. Bellamy related to this set Question 24, [].

The C-compact-open topology on C(X)

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This talk concentrates on the C-compact-open topology on $C(X)$, the set of all real-valued continuous functions on a Tychonov space X and compares this topology with the compact-open topology, the pseudocompact-open topology, the bounded-open topology and the topology of uniform convergence. In the second half, the induced map, as well as the metrizability of this topology, is studied.

The main goal of this paper is to study the properties, such as the countable chain condition, Lindelöf property and second countability of the C-compact-open topology on $C(X)$. But in order to make this study fruitful, these properties of the C-compact-open topology are compared with those of the point-open, pseudocompact-open and compact-open topologies on $C(X)$.

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Separability in Bitopological spaces and selection principles

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In this talk we investigate the selection principles *M*-separability, *R*-separability and *H*-separability in the bitopological context. Specially we will study these properties in function spaces.

Braid Groups in Complex Projective Spaces

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Let M be a manifold and $\mathcal{F}_{\mathrm{k}}(\mathcal{M})$ be *the ordered configuration space* of k distinct points $\{(x_1, \ldots, x_k) \in M^k | x_i \neq x_j,\ i \neq j\}$. There is a proper right action of Σ_k, the symmetric group of order k, on $\mathcal{F}_{\mathrm{k}}(\mathcal{M})$. The orbit space $\mathcal{F}_{\mathrm{k}}(\mathcal{M})/\Sigma_{\mathrm{k}}$ is the *unordered configuration space*, denoted $C_{\mathrm{k}}(\mathcal{M})$. Using the geometrical structure of projective spaces we stratify the configuration spaces $\mathcal{F}_{\mathrm{k}}(\mathbb{CP}^{\mathrm{n}})$ and $C_{\mathrm{k}}(\mathbb{CP}^{\mathrm{n}})$ with complex submanifolds as follows:

$$\mathcal{F}_{\mathrm{k}}(\mathbb{CP}^{\mathrm{n}}) = \coprod_{\mathrm{i}=1}^{\mathrm{n}} \mathcal{F}_{\mathrm{k}}^{\mathrm{i,n}} ,$$

where $\mathcal{F}_{\mathrm{k}}^{\mathrm{i,n}}$ is the ordered configuration space of all k points in $\mathbb{CP}^{\mathrm{n}}$ generating a subspace of dimension i, and

$$C_{\mathrm{k}}(\mathbb{CP}^{\mathrm{n}}) = \coprod_{\mathrm{i}=1}^{\mathrm{n}} C_{\mathrm{k}}^{\mathrm{i,n}} ,$$

where $C_{\mathrm{k}}^{\mathrm{i,n}}$ is the unordered configuration space of all k points in $\mathbb{CP}^{\mathrm{n}}$ generating a subspace of dimension i.

We describe the fundamental groups of ordered and unordered k-point sets in $\mathbb{CP}^{\mathrm{n}}$ generating a projective subspace of dimension i. We apply these to study connectivity of more complicated configurations of points namely, Pappus configuration spaces $\mathcal{P}$ and Desargues configuration spaces $\mathcal{D}$.

On the concept of Σ^1_2-completness

Janusz Pawlikowski

We prove that Selivanovski Σ^1_2-complete sets are Σ^1_2-complete. Namely, if a subset C of a Polish space is such that preimages of C via Selivanovski measurable functions give all Σ^1_2-subsets of the Cantor set then already preimages of C via continuous functions do this. The proof is "classical" (no effective descriptive set theory). Our method also gives a "classical" proof of a theorem of Kechris that Borel Π^1_1-complete sets are Π^1_1-complete. The original proof of Kechris ([]) heavily used effective descriptive set theory and Kechris explicitely asked about a possibility of a “classical” proof.

Coronas of metric spaces

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Let (X,ρ) be a metric space, $x_0 \in X$, X_d be a set X endowed with the discrete topology, βX_d be the Stone-Čech compactification of X_d. We identify βX_d with the set of all ultrafilters on X and denote by $X^\#$ the set of all ultrafilters on X whose members are unbounded subsets of X. A subset A is bounded if there exists $n \in \omega$ such that $A \subseteq B(x_0,n)$ where $B(x_0,n) = \{x \in X : \rho(x_0,x) \leqslant n\}$.We assume that (X,ρ) is unbounded so $X^\#$ is a non-empty closed subset of βX_d.

Given any $p,q \in X^\#$, we say that p,q are *parallel* (and write $r \parallel q$) if there exists $n \in \omega$ such that, for every $P \in p$, we have $B(P,n) \in q$ where $B(P,n) = \cup_{x\in P} B(x,n)$, and denote by $\sim$ the smallest (by inclusion) closed (in $X^\# \times X^\#$) equivalence on $X^\#$ such that $\parallel \subseteq \sim$. The quotient $\check{X} = X^\# / \sim$ is called the *corona* of X. In the case of proper metric space (= each closed ball is compact) $\check{X}$ is homeomorphic to the Higson's corona of X.

Let (X,ρ) be an unbounded metric space, $p,q \in X^\#$. Then

1. $p \sim q$ if and only if $h^\beta(p) = h^\beta(q)$ for every slowly oscillating function $h : (X,\rho) \to [0,1]$ (h is *slowly oscillating* if, for any $n \in \omega$ and $\varepsilon > 0$, there exists a bounded subset V of X such that,$\mathit{diam}\ h(B(x,n)) < \varepsilon$ for every $x \in X \setminus V$);

2. every countable discrete subset of $\check{X}$ is C^*-embedded;

3. if (X,ρ) is ultrametric then, under CH, $\check{X}$ is homeomorphic to ω^*;

4. if (X,ρ) is of bounded geometry then $\check{X}$ contains a weak P-point.

Topologies on the group of invertible transformations

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Let G be a group of all invertible transformations of the [0,1] which are nonsingular respect to Lebesgue measure. We say that $\phi : [0, \infty) \to [0, \infty)$ is an Orlicz function if it is convex and $\phi(x) = 0 \Leftrightarrow x = 0$. We say that an Orlicz function ϕ satisfies the condition Δ' globally iff $(\exists c > 0)(\forall x,y \in [0, \infty))\ (\phi(xy) \le c\phi(x)\phi(y))$.

Definition *Let $h : [0, \infty) \to [0, \infty)$ be a Borel measurable function and $\tau \in G$. Let $\omega_\tau = \frac{d\left(m\circ\tau^{-1}\right)}{dm}$ be the Radon-Nikodym derivative respect to a Lebesgue measure m. We introduce an operator $T^{(h)}_\tau : L^0(m) \to L^0(m)$ by the formula $T^{(h)}_\tau(f) := \left(f \circ \tau^{-1}\right)(h \circ \omega_\tau)$ for $f \in L^0(m)$, where $L^0(m)$ stands for the set of all real m–measurable functions*

Definition *Let ϕ be an Orlicz function which satisfies the condition Δ' globally. Let $h : [0, \infty) \to [0, \infty)$ be Borel measurable. We will denote by $\Theta_{\phi,h}$ the topology on the group G, which is induced from the strong operator topology on $G_h = \{T^{(h)}_\tau : \tau \in G\}$ by the map $T^{(h)}$.*

Theorem *Let ϕ be any Orlicz function. Then coarse topologies $\Theta_{\phi,h}$ on G coincide for all Borel measurable functions $h : [0, \infty) \to [0, \infty)$ which satisfy $h(0) = 0$ and the following two conditions:*

$$(\exists\lambda > 0)(\forall x,y \in [0, \infty))\ |\phi(h(x)) - \phi(h(y))| \le \lambda|x - y| \text{ and}$$

$$(\exists\eta > 0)(\forall x,y \in [0, \infty))\ |\phi^{-1}(x) - \phi^{-1}(y)| \le \eta|h(x) - h(y)|.$$

Neighbourhood operators on categories

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On a given topological space, the notion of closure operator – precisely the Kuratowski closure operator – and that of neighbourhood system are equivalent in a sense that they define each other and give rise to the same data of open sets. During the last two decades, the concept of closure operator has been extended to suitable general categories providing "topological insights" and extending results obtained in the point-set setting. Currently this brings an economy of effort, considerable insight and organization to General Topology.

However, notions such as convergence and properties such as the weak separation axioms cannot be smoothly extended as they are more naturally associated to the notion of neighbourhood and that of open sub-objects. Roughly speaking the unavailability of points and complement makes it that "open" and "closed" are no more dual notions.

We shall present in this lecture a set of axioms defining neighbourhood systems on a suitable category. Few properties shall be given and illustrated through various examples.

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Continuity in groups

Evgenii Reznichenko

Recall that a map $f : X \to Y$ of topological spaces is said to be quasi-continuous if, for any $x \in X$ and any open sets $W \ni f(x)$ and $U \ni x$, there exists a nonempty open set $V \subset U$ for which $f(V) \subset W$. Let G be a group, and let τ be a topology on G. The group with topology (G,τ) is called a *semitopological group* if all right and left translations are continuous, or, in other words, if multiplication is separately continuous.

Let X be a set, and let $P \subset X \times X$. We set $P_x = \{y \in X : (x,y) \in P\}$ for $x \in X$. A subset P of the square $X \times X$ is said to be *semi-open* if each P_x is open. We say that P is a *semi-neighborhood of the diagonal* if P is semi-open and contains the diagonal $\Delta = \{(x,x) : x \in X\}$ of the square $X \times X$.

defn A space X is said to be *Δ-Baire* if, for any semi-neighborhood of the diagonal $P \subset X \times X$, there exists a nonempty open set $W \subset X$ such that $W \times W \subset P$.

The main result is the following theorem.

* Any Δ-Baire regular semitopological group G with quasi-continuous multiplication is a topological group. *

The class of Δ-Baire spaces includes all locally pseudocompact spaces, Baire p-spaces, Baire Σ-spaces, and products of Cech complete spaces.

* Let G be a semitopological group homeomorphic to a product of discrete spaces. Then G is a topological group. cor*

Reflection spaces

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If X is a set, a product $*$ on X will be called a *reflection product* if it satisfies the following axioms for all $x,y,z \in X$:

1. $x * (x * y) = y$, and

2. $x * (y * z) = (x * y) * (x * z)$.

A space X with a reflection product $*$ that is continuous as a map $X \times X \to X$ will be called a *reflection space*. Every sphere is a reflection space, where we define $x * y$ to be the reflection of y across the hyperplane normal to x.

If X is a reflection space, the product on X induces a reflection product on the homotopy classes of maps from a space into X. For an example application, we note that if f and g are maps from a CW complex into S^n, then $[f] = [g] * [g]$ if and only if f and g are homotopy disjoint. In the case of $\pi_i(X,x_0)$, we have that the map $[g] \to [f] * [g]$ is affine for each $[f]$.

Often, very little effort is required to completely describe a reflection product on a discrete set. For example, let us compute the product on $\pi_n(S^n)$, $n > 0$. Together with the axioms, knowledge of the degree of a reflection and the degree of the antipodal map is sufficient to conclude that

$$\deg(f * g) = 2\deg f - \deg g$$

if n is odd, and

$$\deg(f * g) = -\deg g$$

if n is even.

On the unicoherence of quotient spaces of symmetrics products of continua

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A *continuum* means a compact, connected, metric space not degenerated. Given a continuum X and n natural number, $F_n(X)$ denote the n-th symmetric product of X with the Hausdorff metric. If n,m are two natural numbers with $m < n$, $F_n(X)/F_m(X)$ is the quotient space obtained by shrinking $F_m(X)$ to a point in $F_n(X)$, topologized with the quotient topology. In this talk we study the unicoherence of $F_n(X)/F_m(X)$.

On Topologies Generated by κ-Suslin Sets

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We show that topologies on λ^ω generated by κ-Suslin sets satisfy the Baire Category Theorem. Consequently, Projective Determinacy implies that so are topologies on ω^ω generated by effectively projective sets. Using this we establich some dichotomy theorems concerning σ-compactness of effectively projective sets.

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Intrinsic shape of sets in Morse decomposition

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In a dynamical system with a compact phase space X, the main theorem for shape of attractors [], states an existence of a neighbourhood, U of an attractor K such that Sh(U)=Sh(K). By this result, for a Morse decomposition consisting of disjoint compact invariant sets $\{K_1, K_2, ..., K_n\}$ there is a neighbourhood U_1 of K_1 such that Sh(U_1)=Sh(K_1)

Using the new intrinsic approach to shape from [], for arbitrary Morse decomposition $\{K_1, K_2, ..., K_n\}$, we prove that there exist neighbourhoods U_i of K_i, such that Sh(U_i)=Sh(K_i),$i = 1, 2, ..., n$.

This is a joint work with my student M. Shoptrajanov.

Equations in Groups and the Existence of Nontrivial Group Topologies

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Necessary and sufficient conditions for the existence of a nondiscrete Hausdorff group topology on a group in terms of solution sets of systems of equations in this group are given.

Cardinal sequences of scattered spaces

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If X is a scattered topological space, and α is an ordinal, denote by $I_\alpha(X)$ the αth Cantor-Bendixson level of X. The *cardinal sequence* of X, $SEQ(X)$, is the sequence of the cardinalities of the infinite Cantor-Bendixson levels of X, i.e.

$$SEQ(X) = \langle\ |I_\alpha(X)| : \alpha < ht^-(X)\ \rangle,$$

where $ht^-(X)$, the *reduced height* of X, is the minimal β such that $I_\beta(X)$ is finite.

Theorem *It is relatively consistent with ZFC that 2^ω is arbitrarily large and every sequence $\mathfrak{s} = \langle \mathfrak{s}_\mathfrak{i} : \mathfrak{i} < \omega_2 \rangle$ of infinite cardinals with $s_i \leq 2^\omega$ is the cardinal sequence of some locally compact scattered space.*

Descriptive properties of elements of biduals of Banach spaces

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If E is a Banach space, any element x^{**} in its bidual E^{**} is an affine function on the dual unit ball B_{E^*} that might possess variety of descriptive properties with respect to the weak* topology. We prove several results showing that descriptive properties of x^{**} are quite often determined by the behaviour of x^{**} on the set of extreme points of B_{E^*}, generalizing thus results of J. Saint Raymond and F. Jellett. We also prove several results on relation between Baire classes and intrinsic Baire classes of L_1-preduals which were introduced by S.A. Argyros, G. Godefroy and H.P. Rosenthal on p. 1047 of []. Also, several examples witnessing natural limits of our positive results are presented.

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Covering Properties of Symmetrizable Spaces

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The properties that we are interested in are almost Lindelöf and weakly Lindelöf . Let us consider the following natural questions:

Question *Does there exist a Hausdorff symmetrizable space of cardinality exceeding* 2^ω *which is weakly Lindelöf, or almost Lindelöf?*

We show that the answer is negative for T_1-spaces.

Question *Is every H-closed symmetrizable space metrizable? Is every H-closed symmetrizable space compact?*

We show that the answer is "no."

Question *Is every weakly Lindelöf symmetrizable space hereditarily weakly Lindelöf?*

Question *Is every weakly Lindelöf symmetrizable space Lindelöf? Is every weakly Lindelöf symmetrizable space almost Lindelöf?*

We show that the answer is "no."

Question *Is every weakly Lindelöf symmetrizable space c.c.c.?*

Question *Is every almost Lindelöf symmetrizable space Lindelöf?*

Sequence Selection Properties for Quasi-normal Convergence

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We introduce the sequence selection principles, which contributed to alternative proof of Tsaban – Zdomskyy's Theorem []. That theorem asserts that a perfectly normal space is a QN-space [] if and only if its Borel images in the Baire space are bounded. As a consequence many well-known covering properties and selection principles on a perfectly normal space formulated for Borel covers or Borel functions are equivalent to QN-property.

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Some consistent counterexamples in the theory of D-spaces

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We present a method for constructing interesting hereditarily Lindelöf T_2 spaces that are not D-spaces. E.g., assuming $\diamond$ there is a hereditarily Lindelof T_2 space that is not a D-space, and assuming another a slight strengthening of $\diamond$, there is a hereditarily Lindelöf T_2 space that is not a D-space but is the union of two subspaces both of which are D-spaces. Whether the construction can be modified to produce regular examples is open.

Productively Lindelof Spaces and Selection Principles

Franklin D. Tall

The class of *productively Lindelöf spaces*, i.e. spaces such that their product with every Lindelöf space is Lindelöf, was introduced by Michael and is poorly understood. In particular, Michael's question as to whether countable powers of productively Lindelöf spaces are Lindelöf is still open, although I and my students have made some progress. In a series of papers, some with Aurichi and/or Alas, Junqueira, and Tsaban, I have explored connections of productively Lindelöf spaces with selection principles such as the Menger and Hurewicz properties. Here is a typical result:

Theorem *The Continuum Hypothesis implies that productively Lindelöf spaces are projectively σ-compact, hence Menger and Hurewicz, and hence are D-spaces.*

Markov's problems through the looking glass of Zariski and Markov topologies

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Given a group G, Markov defined an *elementary algebraic subset* X of G to be the solution-set of a one-variable equation over G, i.e. $X = \{x \in G \mid g_1 x^{\varepsilon_1} g_2 x^{\varepsilon_2} \cdots g_n x^{\varepsilon_n} = e_G\}$ for $n \in \mathbb{N}$, $\varepsilon_1, \ldots, \varepsilon_n \in \mathbb{Z}$, and $g_1, \ldots, g_n \in G$. He also defined a subset X to be *unconditionally closed in* G if it were closed in every Hausdorff group topology on G.

These definitions implicitly introduced two T_1 topologies on G, now called the *Zariski topology* $\mathfrak{Z}_\mathfrak{G}$ of G, generated by the elementary algebraic subsets as closed sets, and the *Markov topology* $\mathfrak{M}_\mathfrak{G}$ of G, generated by the unconditionally closed subsets as closed sets (this topology is the intersection of every Hausdorff group topology on G).

One can easily see that $\mathfrak{Z}_\mathfrak{G} \subseteq \mathfrak{M}_\mathfrak{G}$, and Markov himself proved that they coincide if the group G is countable. He asked whether these two topologies always coincide, and Hesse built an example showing that this is not true in general.

The more topologies the group G carries, the coarser $\mathfrak{M}_\mathfrak{G}$ is; while $\mathfrak{M}_\mathfrak{G}$ is discrete if and only if the discrete topology is the only Hausdorff group topology on G (call non-topologizable group such a group). Markov also asked whether an infinite non-topologizable group could exist, and Shelah built the first example of such a group.

I will speak about these and other problems posed by Markov, and recent developments in this area.

Buried points of Peano Continua in the Plane

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For a plane continuum X, let F denote the union of all boundaries of complementary components of X. Then the complement of F is called the set of buried points of X.

In their 2010 paper, Curry and Mayer gave an introduction to buried points in Julia sets and a list of open questions. Subsequently, Curry, Mayer and Tymchatyn began a study of topological properties of plane continua and buried points. In particular, they were interested in the following problem: under what conditions the totally disconnected set of buried points of a plane continuum is 0-dimensional? In this talk, we look at some results in relation to this problem and give an example of a Peano continuum in the plane with weakly one-dimensional, totally disconnected buried set.

On continua on which generic maps have 0-dim. sets of chain-recurrent points.

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Denote by 0-CR the class of compact metric spaces X such that continuous maps $f : X \to X$ with the chain-recurrent set $CR(f)$ of dimension 0 form a dense G_δ subset of the space of all continuous self maps on X (with the uniform convergence). We show that the class 0-CR contains all locally connected curves, polyhedra, Menger compacta, the Hilbert cube and many non-locally connected continua. The results extend similar theorems by many authors for homeomorphisms or continuous maps on manifolds.

Ordinal remainders of classical ψ-spaces

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Let ω denote the set of natural numbers, and $\mathfrak{t}$ the tower number, i.e., the smallest cardinality of tower in $[\omega]^\omega$. We prove: For every ordinal $\lambda < \mathfrak{t}^+$, there exists $\mathcal{M} \subset [\omega]^\omega$, an infinite maximal almost disjoint family of infinite subsets of the natural numbers (MADF), such that the Stone-Čech remainder, $\beta\psi \setminus \psi$, of the ψ-space, $\psi = \psi(\omega,\mathcal{M})$, is homeomorphic to $\lambda + 1$ with the order topology. This generalizes a result credited to S. Mrówka by J. Terasawa which states that there is MADF $\mathcal{M}$ such that $\beta\psi \setminus \psi$ is homeomorphic to $\omega_1 + 1$. We construct our MADF from an ascending mod-finite ordered chains of infinite subsets of ω, ordered by almost inclusion.

[32] The first author was partially supported by the NSF

Completion in supernear spaces

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In this paper we consider supernearness that is a common generalization of supertopologies and nearness in the sense of Doitchinov and Herrlich, respectively. As observed by Tozzi and Wyler supertopologies can be equivalently described by the nearness of bounded to arbitrary sets. Hence, this gives us the opportunity for defining B-near collections as those which are contained in the clan of sets near to B. In consequence this lead us to a so called conic super- near operator, and conversely each of such special operator induces back our given nearness between sets. On the other hand nearness collections in the sense of Herrlich are determined by a supernear operator as elements occuring in the intersection of corresponding images of the operator. Then we define completeness on supernearness in a natural way that followed by a completion process on symmetric supernear spaces. It turns out that in this broader realm Herrlich's completion of nearness can be regarded as special case.

Waraszkiewicz spirals revisited

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We present a simple proof of the existence of an uncountable family of plane continua no one of which can be continuously mapped onto any other. This family is a remake of famous Waraszkiewicz's spirals. We study compactifications of a ray with the remainder a simple closed curve. We give a necessary and sufficient condition for continuous comparability of two such continua. From this we construct an uncountable family of such continua.

Embedding cones in hyperspaces

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Given a continuum X, let $C(X)$ be the hyperspace of subcontinua of X and $Cone(X)$ be the topological cone of X.We say that a continuum X is cone-embeddable in $C(X)$ provided that there is an embedding h from $Cone(X)$ into $C(X)$ such that $h(x,0) = \{x\}$ for each x in X.

In this talk, we present some results concerning cone-embeddable in $C(X)$ continua. Mainly on continua such as dendroids, compactifications of the ray, finite graphs and hyperspaces.

[33] The first author was partially supported by UNAM

Box Products of non-Metrizable compacta

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Suppose X is a space. Give B, the product of countable many copies of X, the box topology. The well-known open problem asks whether X compact metric implies B is normal. Beginning with Rudin's assumption of CH, consistent affirmative answers have also been presented by van Douwen, Kunen, Roitman, and the author. Much less is known for compact spaces which are merely first countable or scattered, even when X is the lexicographic ordered square or Fort's space. We present some new results.

The σ-ideal generated by zero-dimensional Z-sets in the Hilbert cube

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We show that the σ-ideal of 0-dimensional Z-sets on the Hilbert cube $[0,1]^{\omega}$ has the same cardinal characteristics as the ideal of first category sets. These results settle, in particular, the known problem (e.g. stated at the webpage of prof. M. Csörnyei, []) whether the Hilbert cube can be cover by the same number of copies of the Cantor set which is sufficient to cover the interval [0,1]. Methods used in the proofs are, among others, a translation of certain results of Bartoszyński for the Baire space to some special topological groups and using the topological structure of certain hyperspaces.

On β-favorability of the strong Choquet game

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In the *strong Choquet game* $Ch(X)$ two players, α and β, take turns in choosing objects in a topological space X: β starts, and always chooses an open set V and a point $x \in V$, then α responds by just an open set U such that $x \in U \subseteq V$. After countably many rounds, α wins the game if the intersection of the chosen open sets is nonempty; otherwise, β wins. *Telgársky* asked whether the existence of a winning strategy for β in $Ch(X)$ is equivalent to the existence of a nonempty W_δ subset of X, which is of the 1st category in itself. It will be answered in the positive in 1st countable R_0 spaces; some related counterexamples will be given in the non-1st countable setting. Using non-β-favorability of $Ch(X)$, new results concerning hereditary Baireness and products of Baire spaces will be discussed.

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